Sveučilište u Dubrovniku
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Logika, skupovi i diskretna matematika

Exercise 5

The assignments are due to 08.01.2007.

Tutorial 5.1

1. Use mathematical induction to prove the equality $1 + 3 + 5 + \cdots + (2n - 1) = n^2$ for every positive integer n.

2. Use mathematical induction to prove the equality $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(n+2)}{6}$ for every positive integer n.

3. Use mathematical induction to prove the inequality $2n + 1 \leq 2^n$ for every $n = 3, 4, \ldots$.

4. Use mathematical induction to prove that $7^n - 1$ is divisible by 6 for all $n \geq 1$.

Tutorial 5.2

1. Write out a proof (using induction on n) of the fact that of $A_1, A_2, \ldots, A_n$ are mutually disjoint sets, then
$$|A_1 \cup A_2 \cup \cdots \cup A_n| = |A_1| + |A_2| + \cdots + |A_n|.$$

2. How many national flags can be constructed from three equal vertical strips, using the colors red, white, blue, and green?

3. A committee of nine people must elect a chairman, secretary and treasurer. In how many ways can this be done?

4. Calculate the total number of permutations of $[1, 6]$ which satisfy $\sigma^2 = \text{id}$ and $\sigma \neq \text{id}$.

Tutorial 5.3

1. Calculate the coefficient of
   (a) x^6 in $(1 + x)^{12}$,
   (b) a^3b^7 in $(a + b)^{10}$,
   (c) a^4b^6 in $(a^2 + b)^8$.

2. In some lottery game, a person was required to choose six numbers (in any order) among 44 numbers. In how many ways can this be done? In how many ways can it be done when choosing six numbers among 48 numbers?

3. Suppose we have a shipment of 50 microprocessors of which four are defective.
   (a) In how many ways can we select a set of four microprocessors?
   (b) In how many ways can we select a set of four nondefective microprocessors?
   (c) In how many ways can we select a set of four microprocessors containing exactly two defective microprocessors?
   (d) In how many ways can we select a set of four microprocessors containing at least one defective microprocessor?

Homework assignment 5.1 **6 points**

1. Use mathematical induction to prove the equality $1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ for every positive integer n.
2. Use mathematical induction to prove the equality

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

   for every positive integer n.
3. Use mathematical induction to prove that $11^n - 6$ is divisible by 5 for all $n \geq 1$.

Homework assignment 5.2 **5 points**

1. Keys are made by cutting incisions of various depths in a number of positions on a blank key. If there are eight possible depths, how many positions are required to make one million different keys? (Hint: to facilitate the calculation use the fact that 2^{10} is slightly greater than 10^3.)
2. In the usual set of dominos each domino may be represented by the symbol $[x \mid y]$, where x and y are members of the set $\{0, 1, 2, 3, 4, 5, 6\}$. The numbers x and y may be equal. Explain as to why the total number of dominos is 28 rather than 49.
3. How many five-digit telephone numbers have a digit which occurs more than once?

Homework assignment 5.3 **4 points**

1. Calculate the coefficient of
   (a) x^5 in $(1+x)^{11}$,
   (b) a^6b^6 in $(a^2+b^3)^5$,
   (c) x^3 in $(3+4x)^6$.
2. In a class of 67 computer science students, 47 can read French, 35 can read German and 23 can read both languages. How many can read neither language. If, furthermore, 20 can read Russian, of whom 12 also read French, 11 read German also and 5 read all three languages, how many can read any of the three languages?