Complements of Database Views: Uniqueness and Optimality Issues

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- However, the results are not limited to the relational model in any way.

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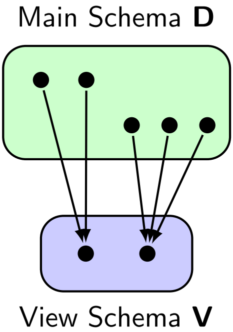

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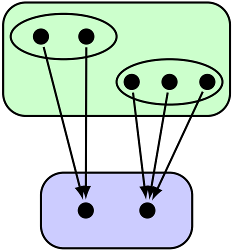

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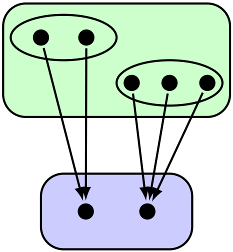

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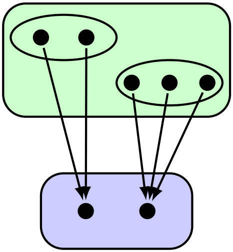

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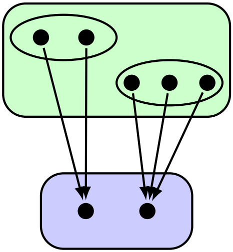

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- For the purposes of this work, views with identical congruences are considered to be *isomorphic*.

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- $\text{Constr}(\mathbf{V}) = \{\text{Card}(R[B]) \leq \text{Card}(R[A])\}$.

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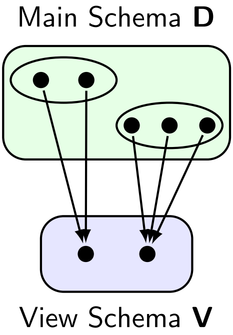

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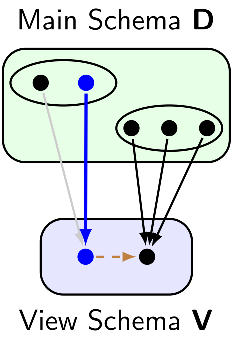

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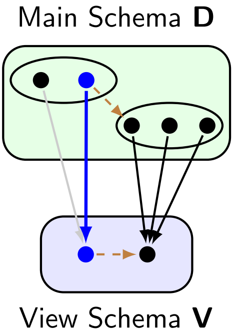

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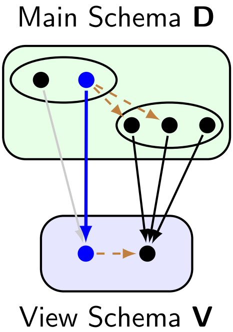

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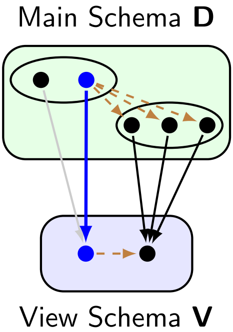

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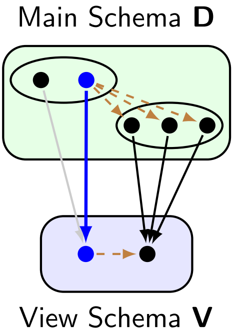

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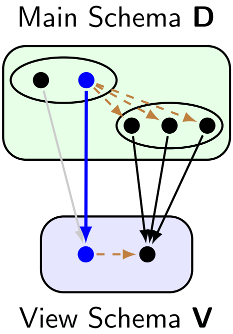

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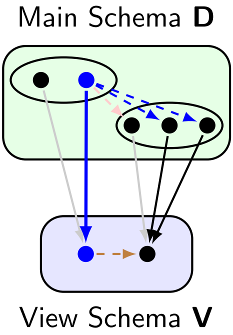

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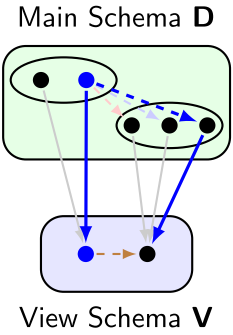

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- Support for this approach is the main focus of this presentation.

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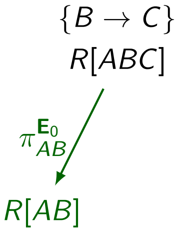

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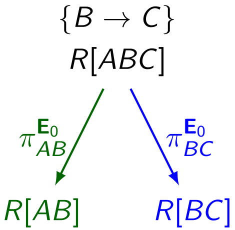

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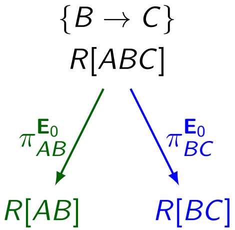

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Observation: These three conditions are in general independent of one another.

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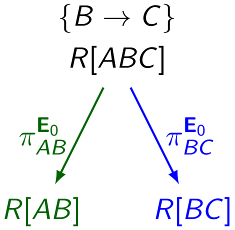

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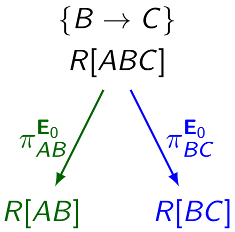

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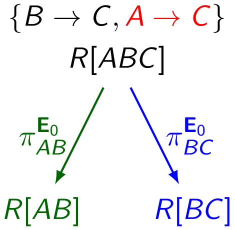

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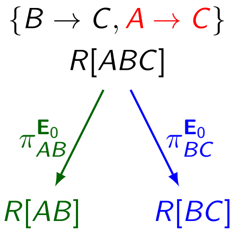

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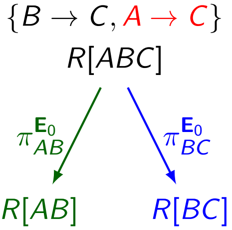

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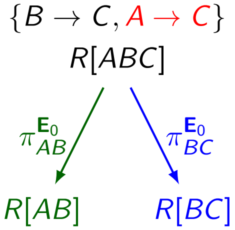

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- With the addition of $A \rightarrow C$, a cover of the dependencies no longer embeds in the views, so these dependencies cannot be checked on a view-by-view basis.

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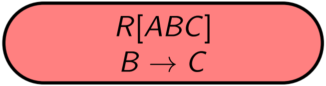

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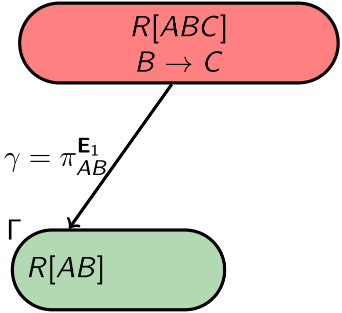

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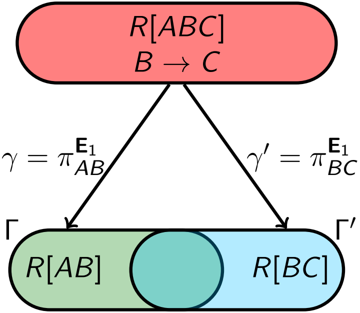

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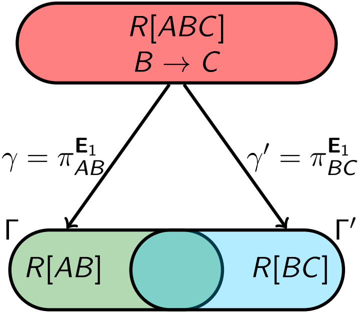

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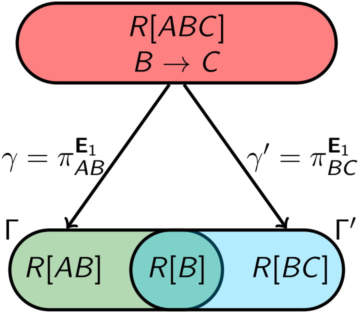

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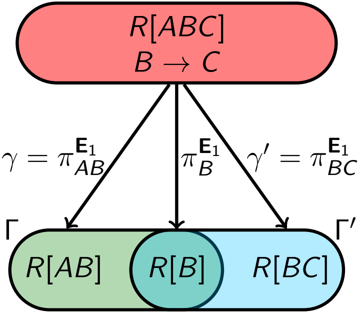

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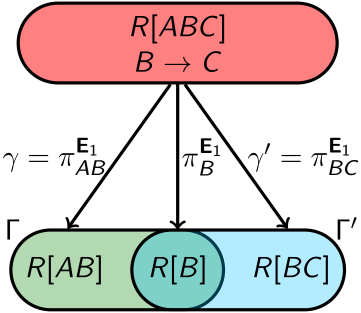

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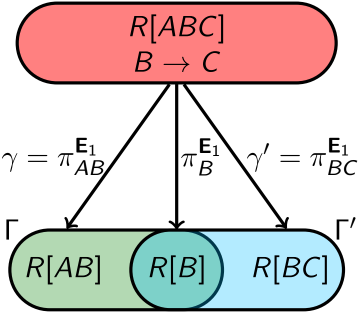

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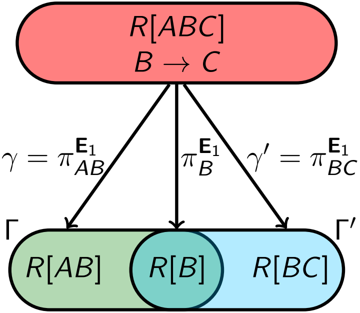

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Theorem: Commuting congruences identifies precisely the conditions under which state invariance holds. $\square$

- A complement with commuting congruences is called a *meet complement*.

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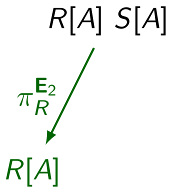

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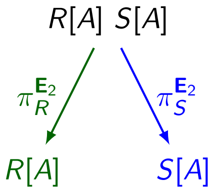

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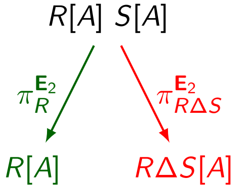

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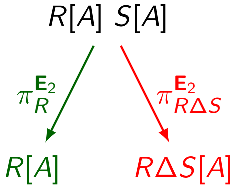

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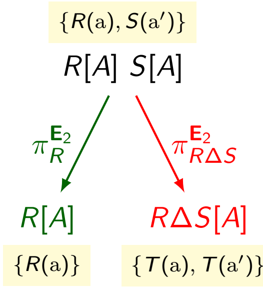

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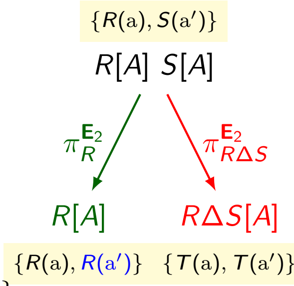

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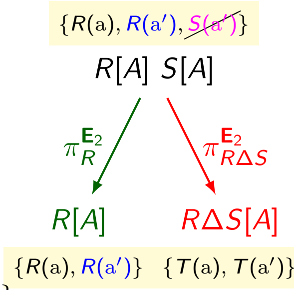

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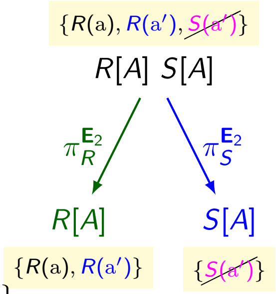

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- But note that *update-set invariance* is satisfied — both complements support all view updates.

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Order-based update: An update which is representable as a composition of insertions and deletions.

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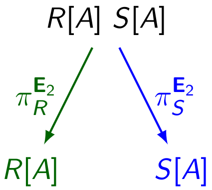

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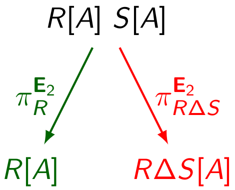

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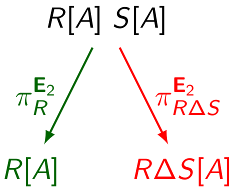

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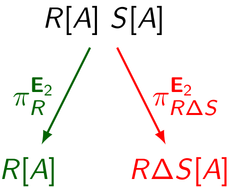

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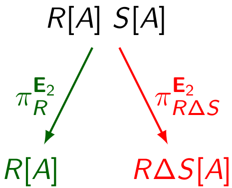

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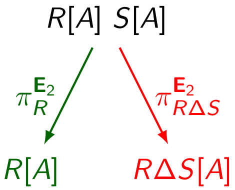

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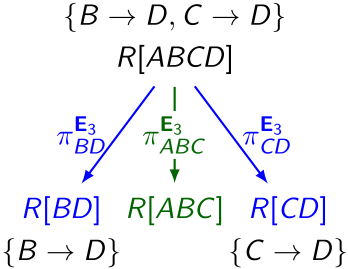

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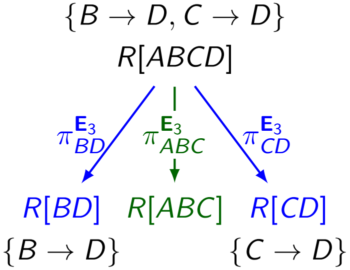

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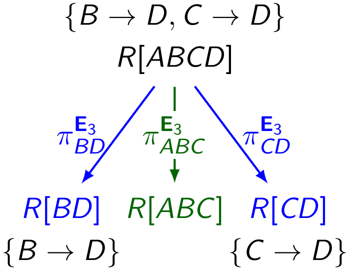

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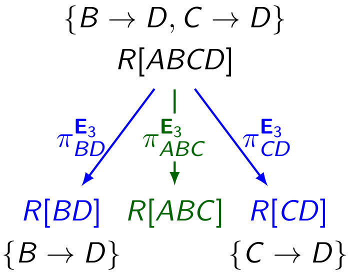

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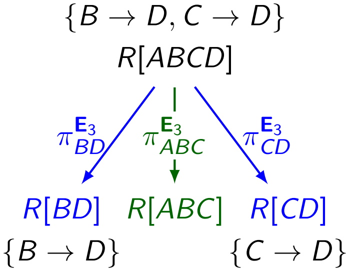

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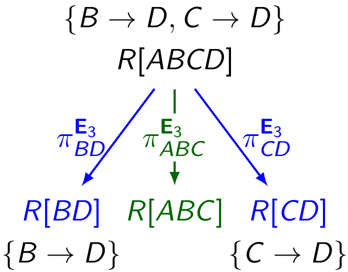

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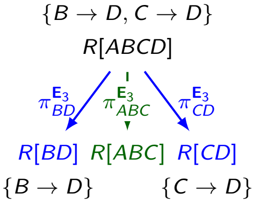

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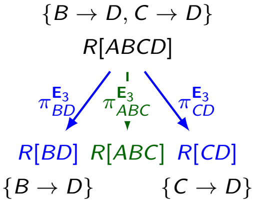

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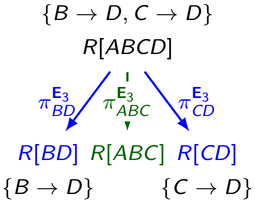

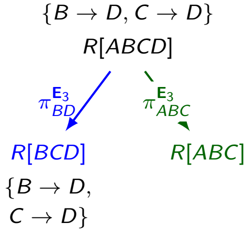

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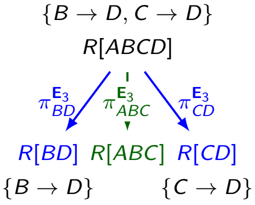

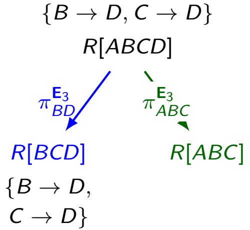

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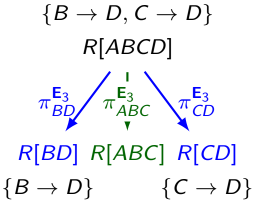

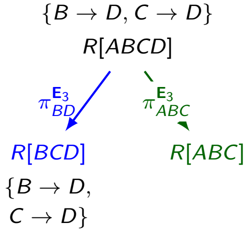

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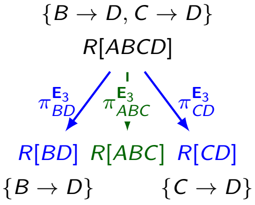
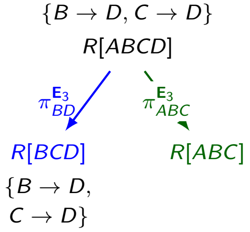
- Both $\Pi_{D}^{\mathbf{E}_3}$ and $\Pi_{CD}^{\mathbf{E}_3}$ are minimal complements of $\Pi_{ABC}^{\mathbf{E}_3}$ relative to the projections $\Pi\text{-Views}\langle\mathbf{E}_3\rangle$.
- Thus, neither can be optimal.
- $\Pi_{BCD}^{\mathbf{E}_3}$ is a complement which is not minimal relative to $\Pi\text{-Views}\langle\mathbf{E}_3\rangle$.
- But $\Pi_{BCD}^{\mathbf{E}_3}$ is an optimal *meet* complement amongst projections.
- If *state invariance* is desired, $\Pi_{BCD}^{\mathbf{E}_3}$ is the best which can be achieved.
  - *Update-set independence comes as a bonus*, but for a smaller set of updates than supported by $\Pi_{BD}^{\mathbf{E}_3}$ or $\Pi_{CD}^{\mathbf{E}_3}$ as complements.





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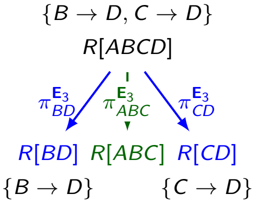

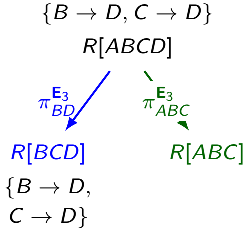

The Context of $\bigvee\Pi$-Views

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$$\{B \to D, C \to D\}$$

$$R[ABCD]$$

$$\downarrow \pi^{\mathbf{E}_3}_{ABC}$$

$$R[ABC]$$

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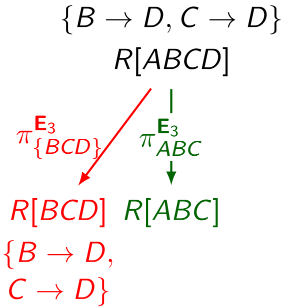

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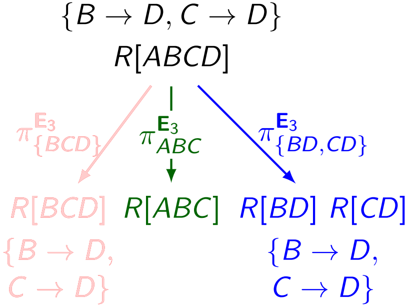

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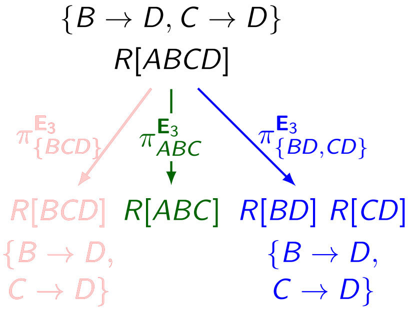

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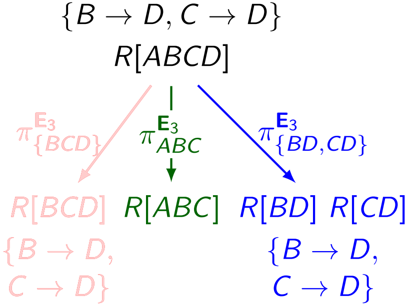

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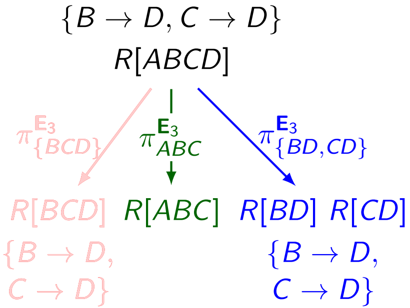

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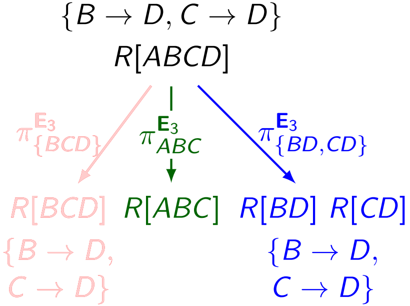

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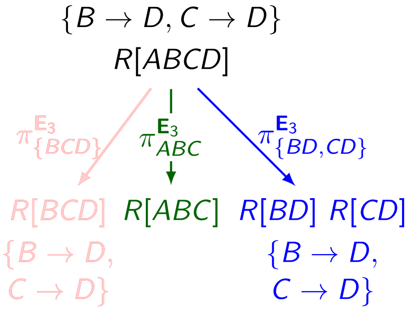

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Notation: $\bigvee\Pi$-Views$\langle \mathbf{D} \rangle$ denotes the set of all $\bigvee\Pi$ views of $\mathbf{D}$.

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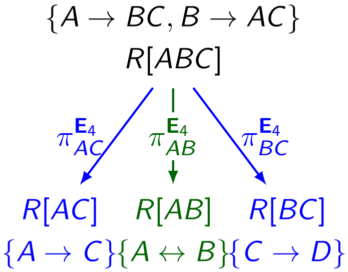

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Context:
- Universal relational schema $\mathbf{D} = (R[\mathbf{U}], \mathcal{F})$; $\mathcal{F}$ = FDs.
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Corollary If $\mathcal{F}$ does not contain any nontrivial FD-equivalences ($\mathbf{Y} \neq \mathbf{Z}$), then $\sqcap^{\mathbf{D}}_{\{\mathbf{W}_1,\mathbf{W}_2,\ldots\mathbf{W}_m\}}$ has a unique optimal meet $\bigvee\!\sqcap$-complement. $\square$

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- The view $\Pi^{\mathbf{E}_6}_{\{AB_{11}B_{12}B_2,\ CD_1D_2\}}$ has four distinct meet complements:

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- Certain useful cases of non-unary IDs can also be handled.

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Further Directions:

- Pursue a more general theory of optimal meet complements which is not dependent upon specific constraints and the relational model.