Optimal Complements for a Class of Relational Views

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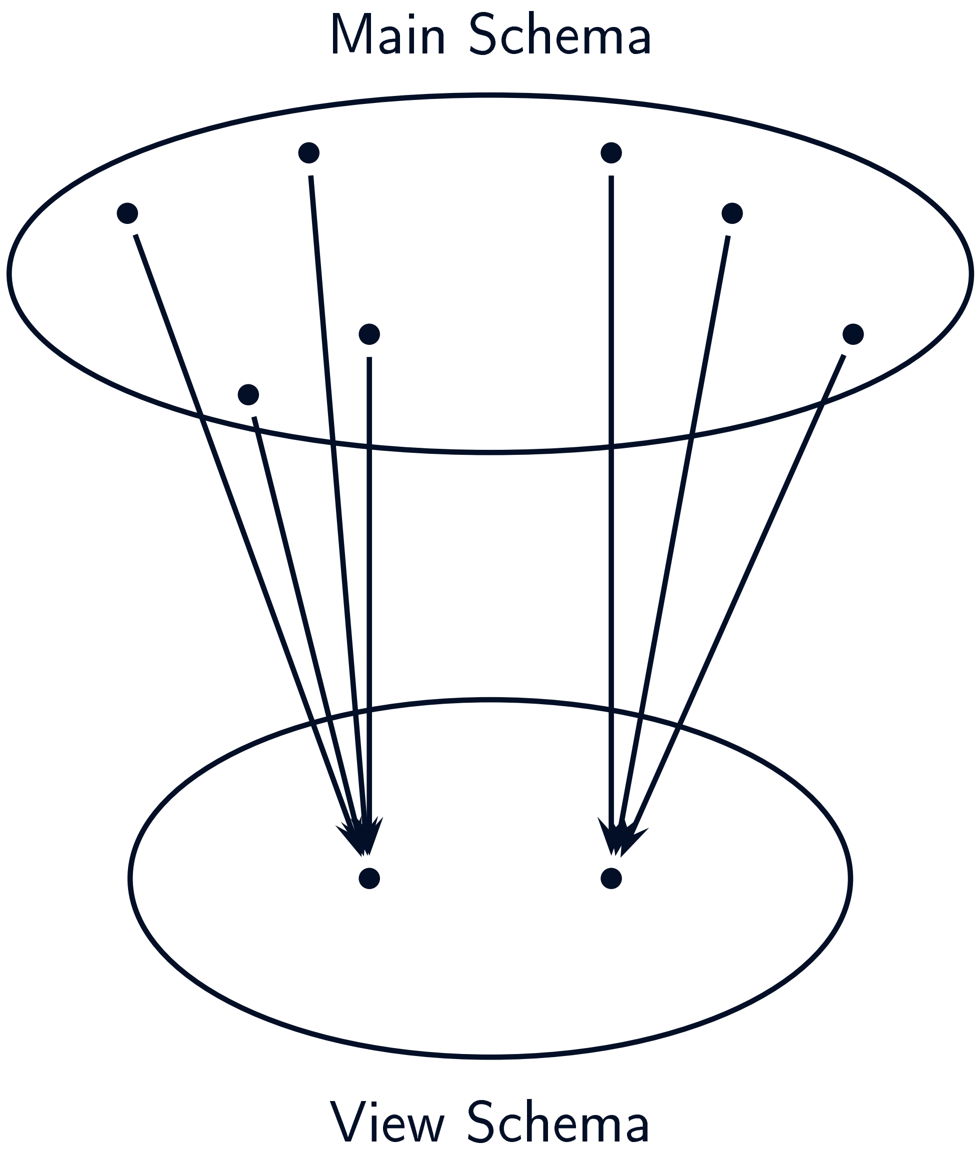

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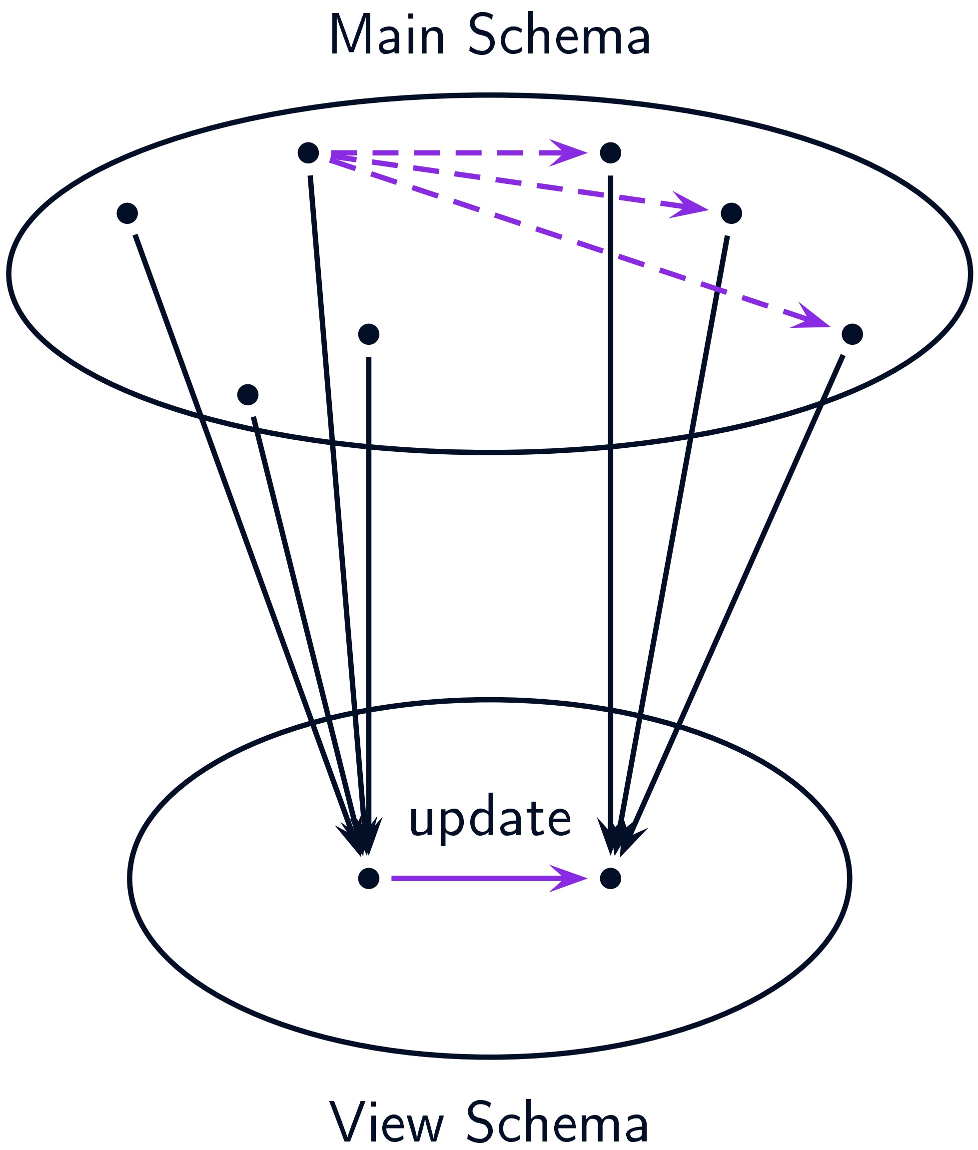

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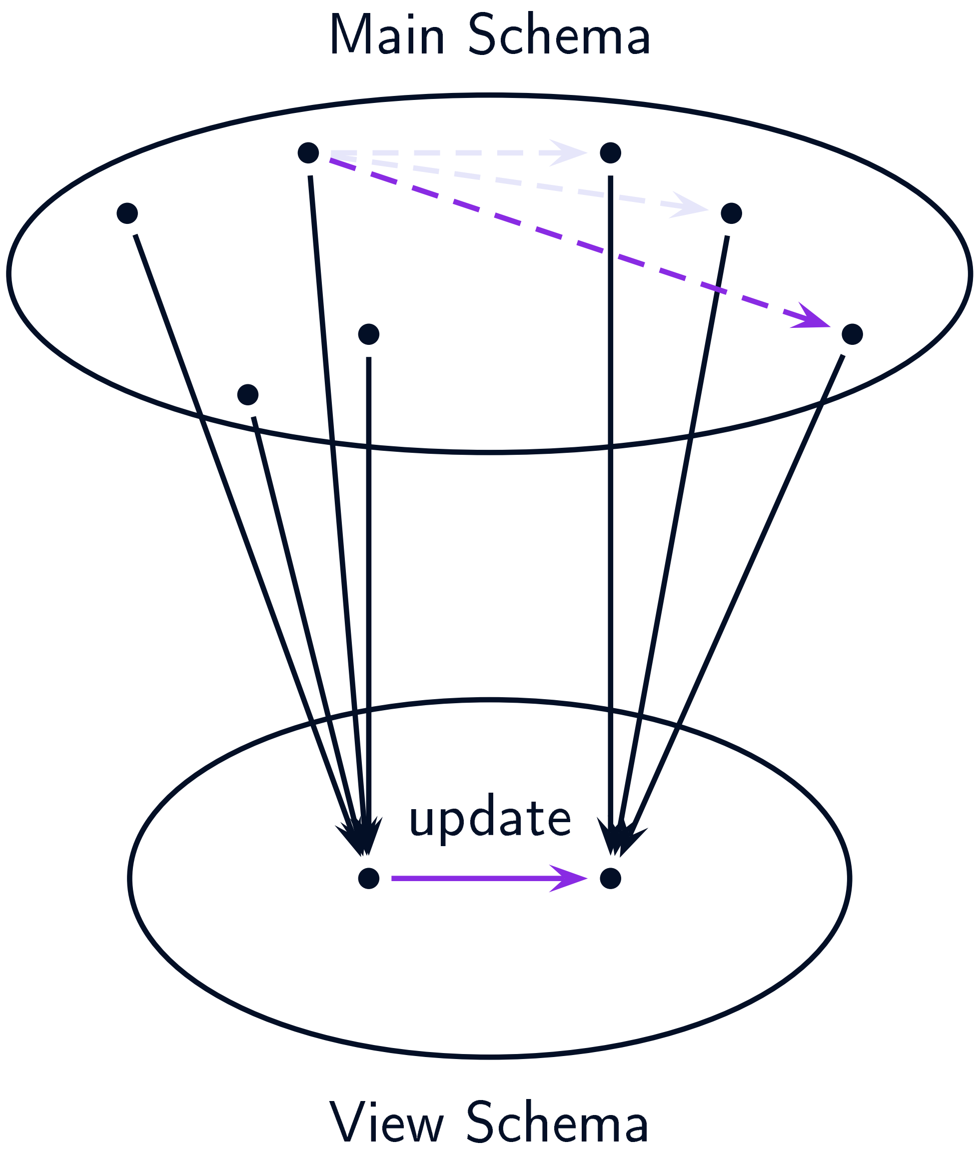

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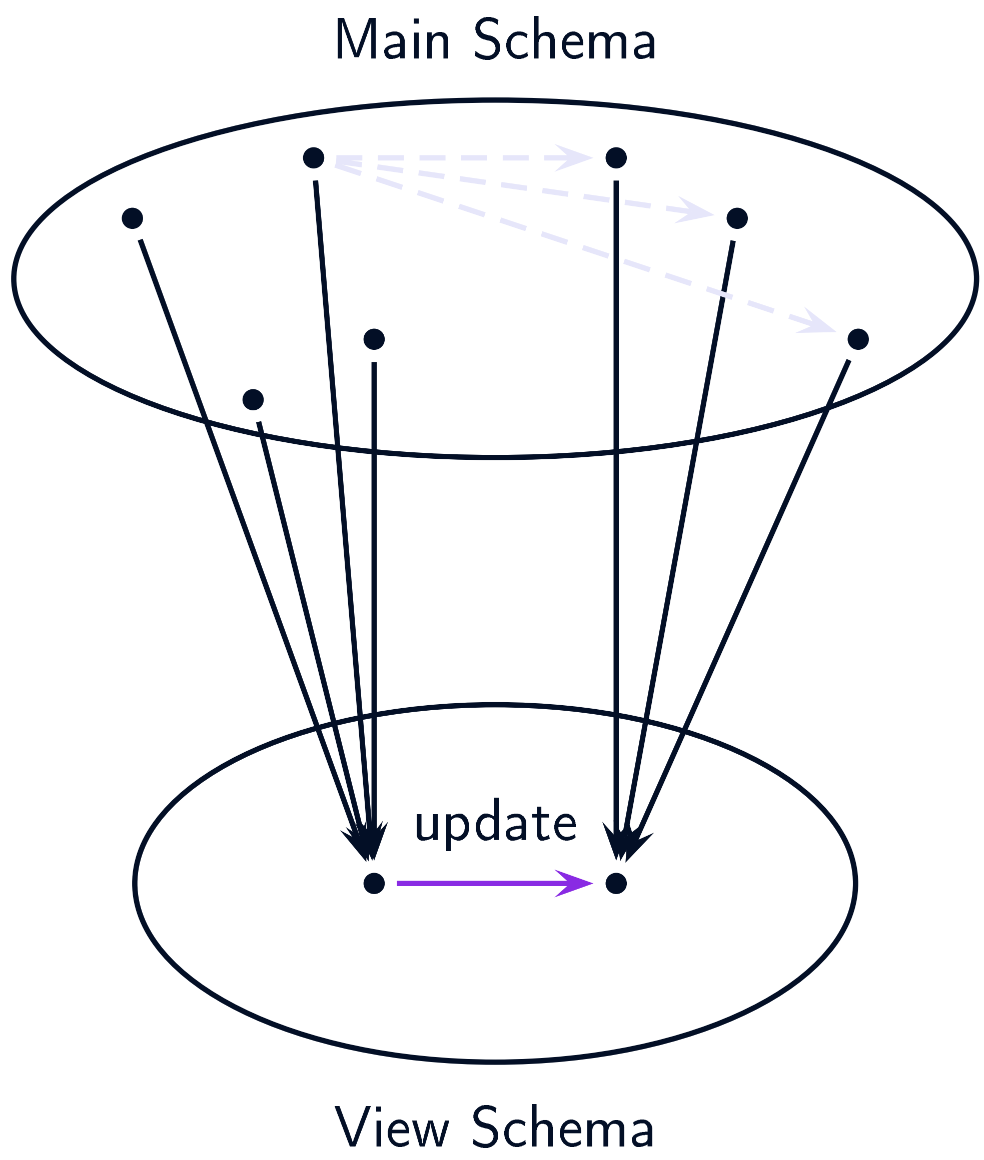
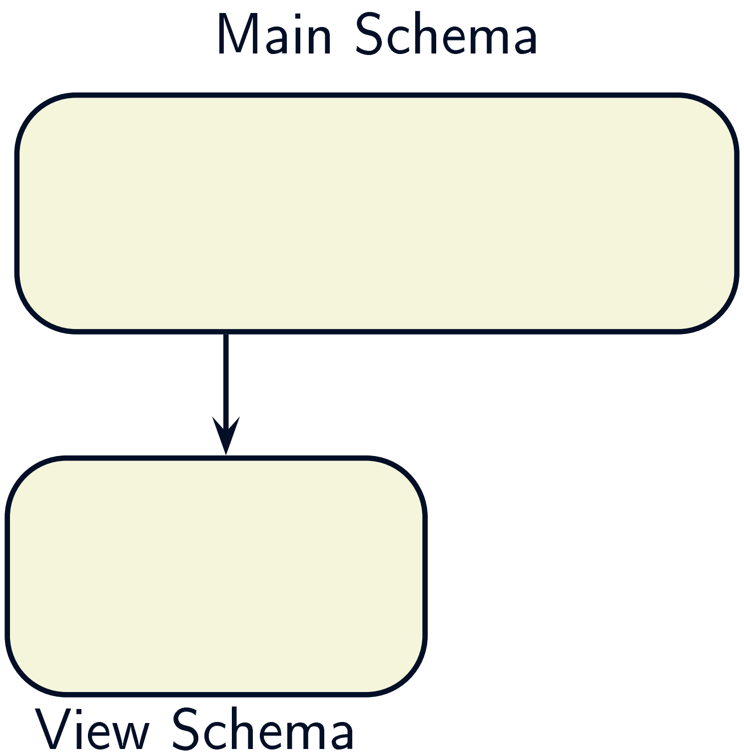
- With a reasonable definition of suitability, it may not be the case that every view update has a suitable translation.



The Gold Standard — the Constant-Complement Strategy



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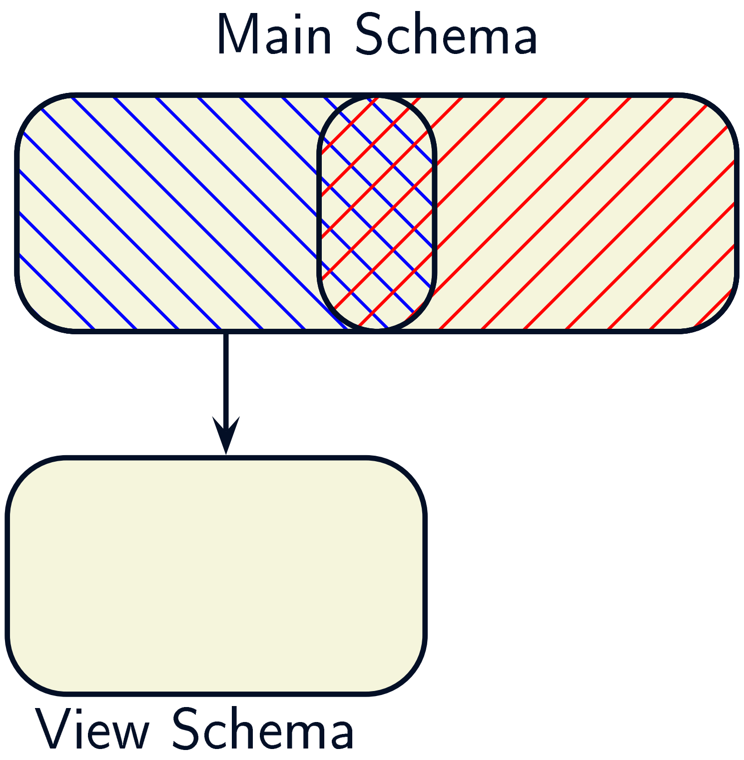

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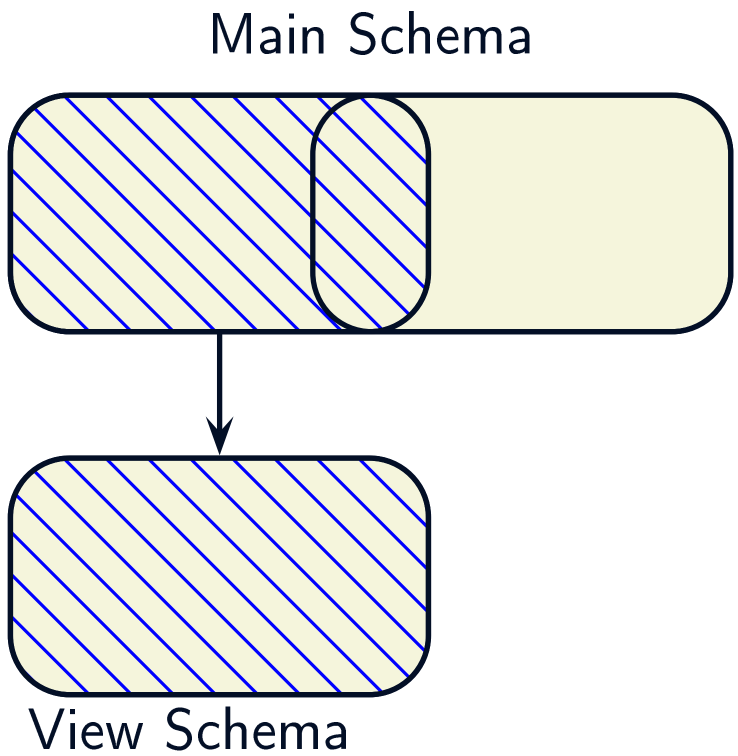

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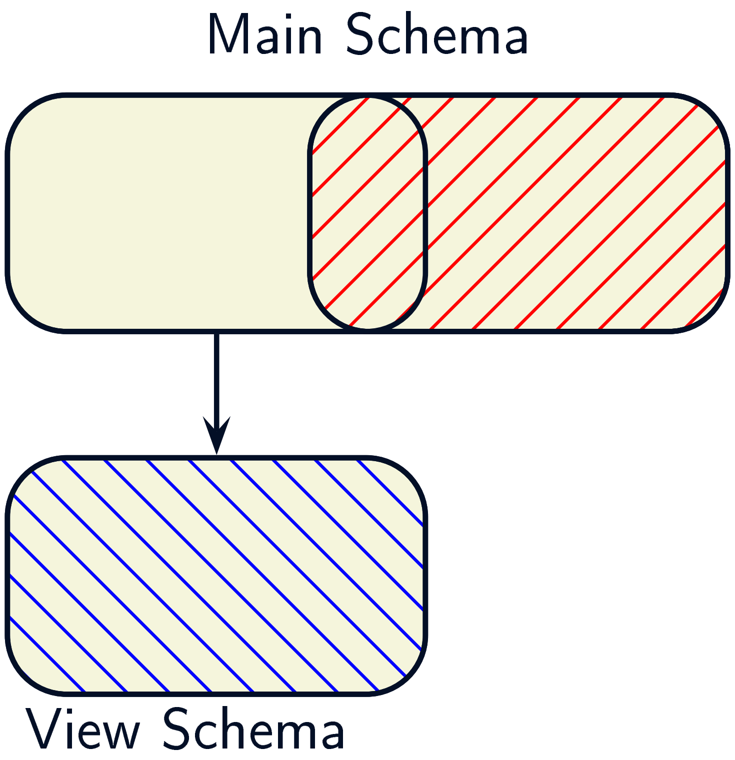

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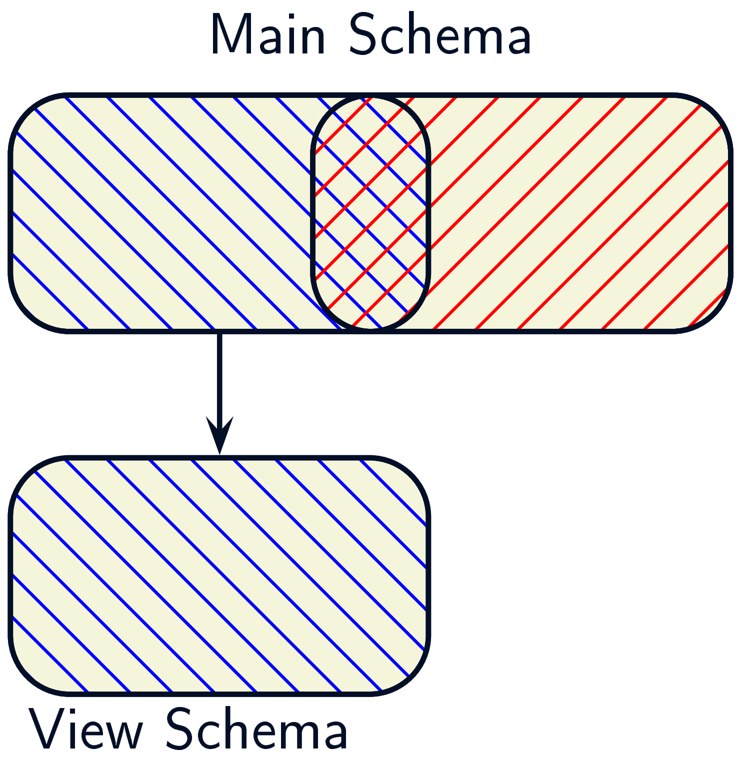

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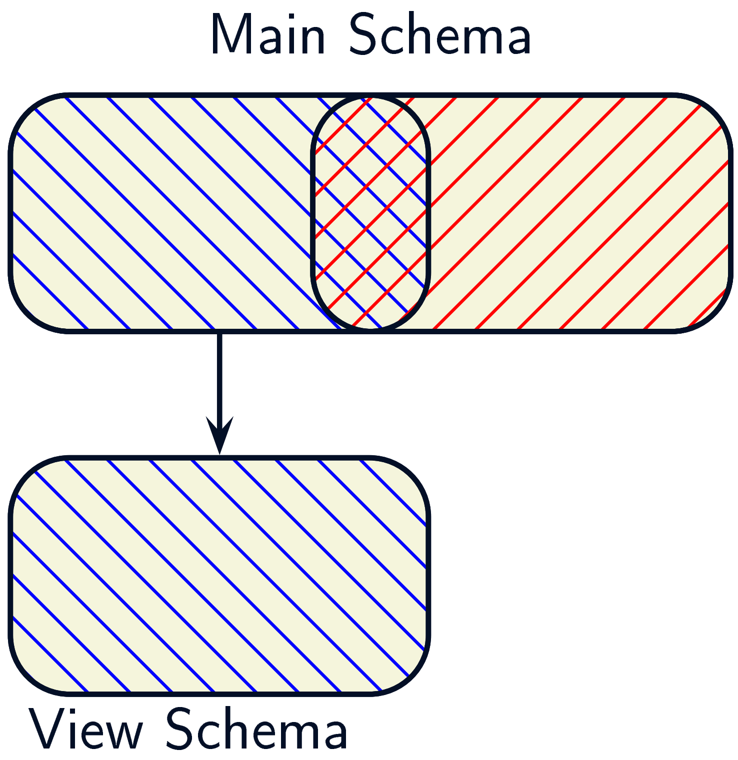

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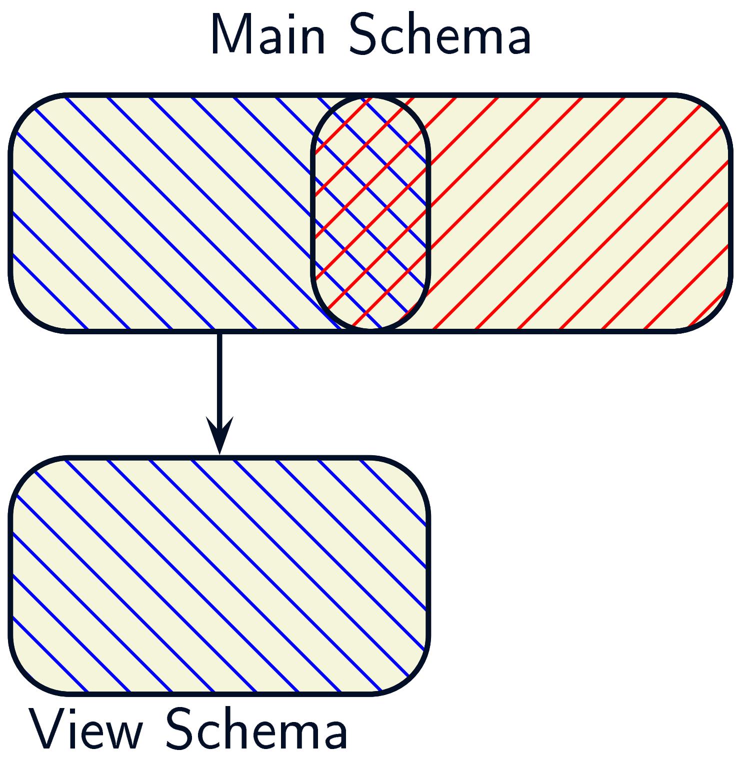

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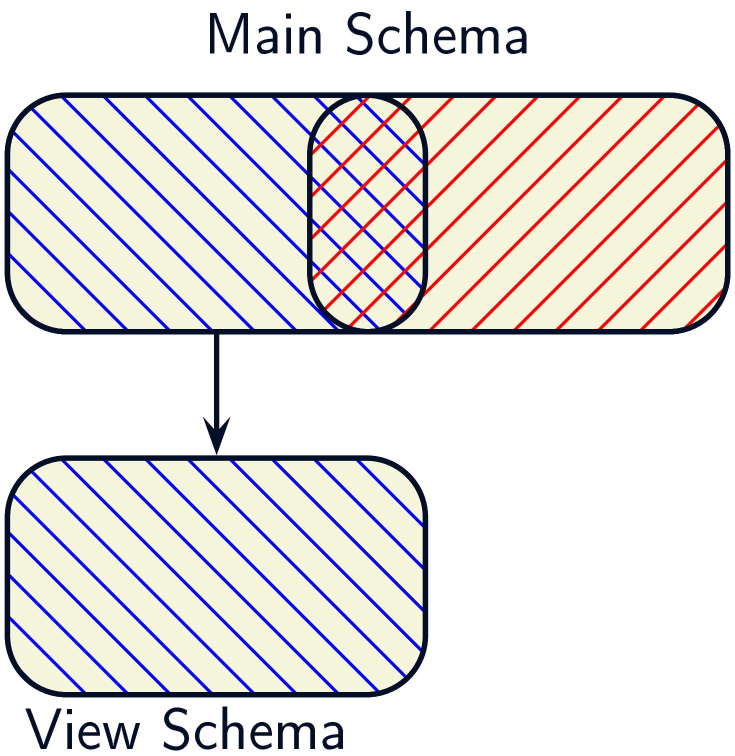
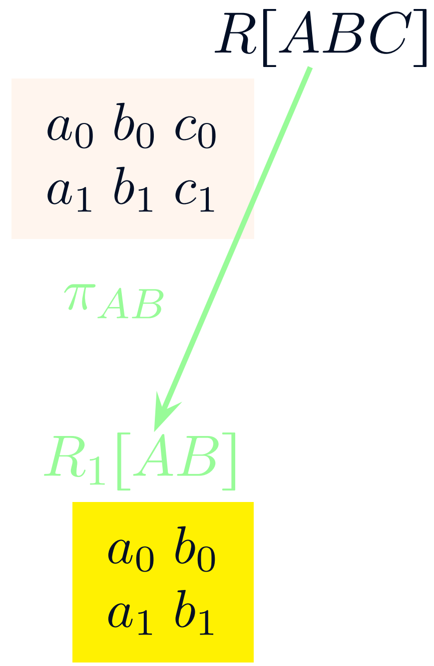


- It can be shown [Hegner 03 AMAI] that this strategy is precisely that which avoids all *update anomalies*.
- However, this is complicated by the *complement uniqueness problem*.
- Some examples will help illustrate these ideas.

- Consider the classical example to the right.

Main Schema $\mathbf{E}_1$
Constraint: $\bowtie [AB, BC]$



View Schema
$\mathbf{W}_{AB}$

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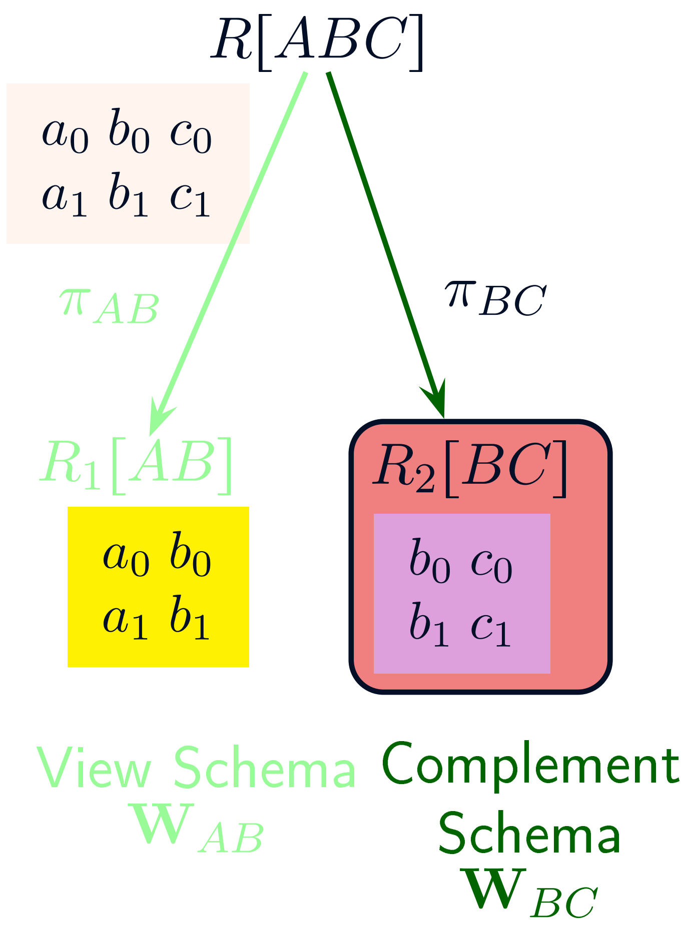

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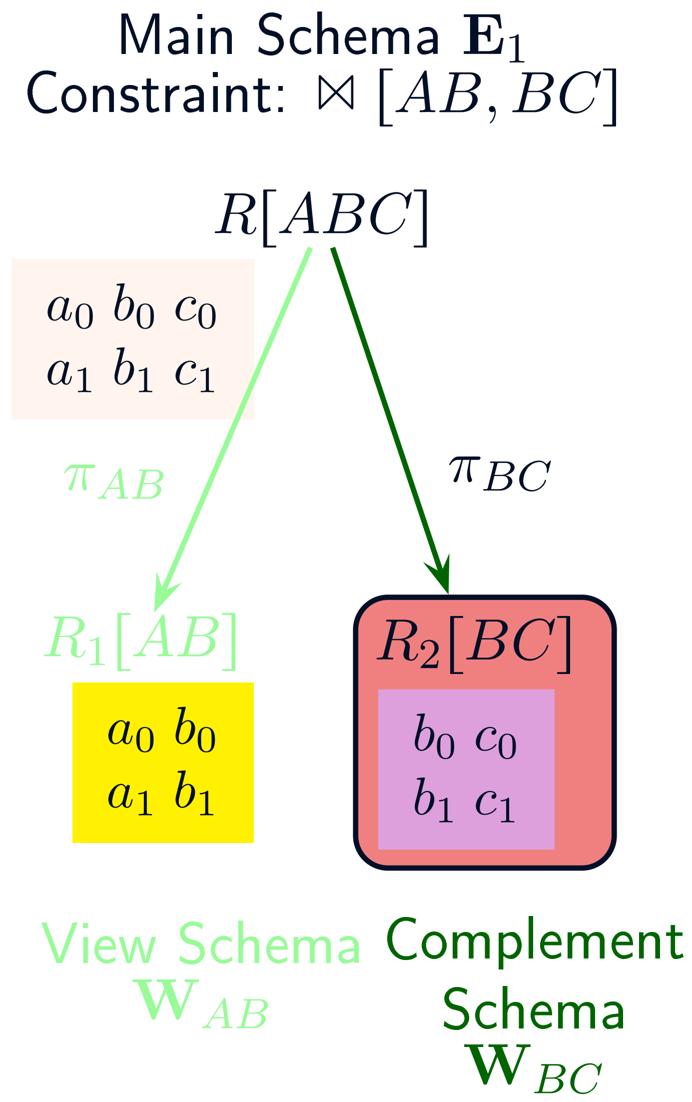

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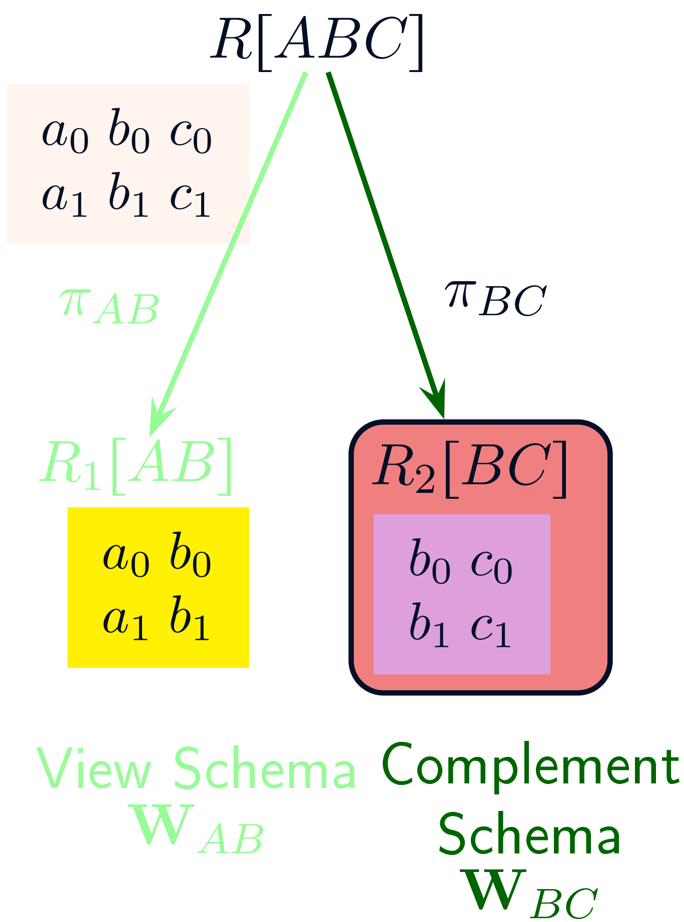

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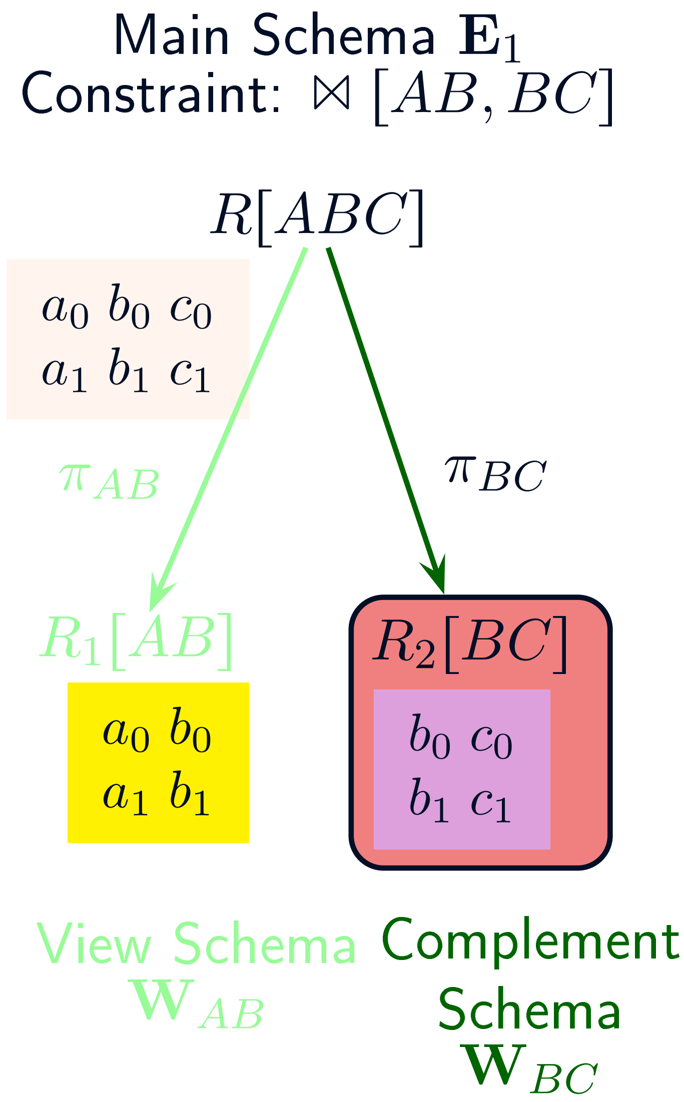

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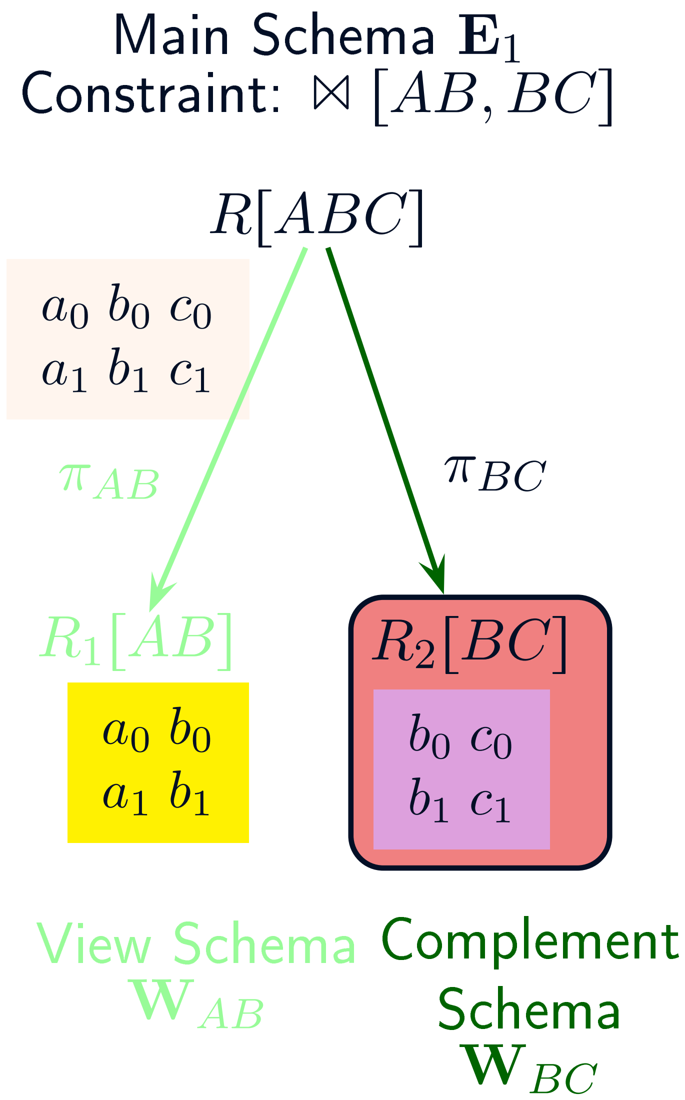
- The view which is the projection on B is the *meet* of $\mathbf{W}_{AB}$ and $\mathbf{W}_{BC}$, and is precisely that which must be held constant under a constant-complement update.



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No dependencies

$R[A]$ $S[A]$

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a_0
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a_2

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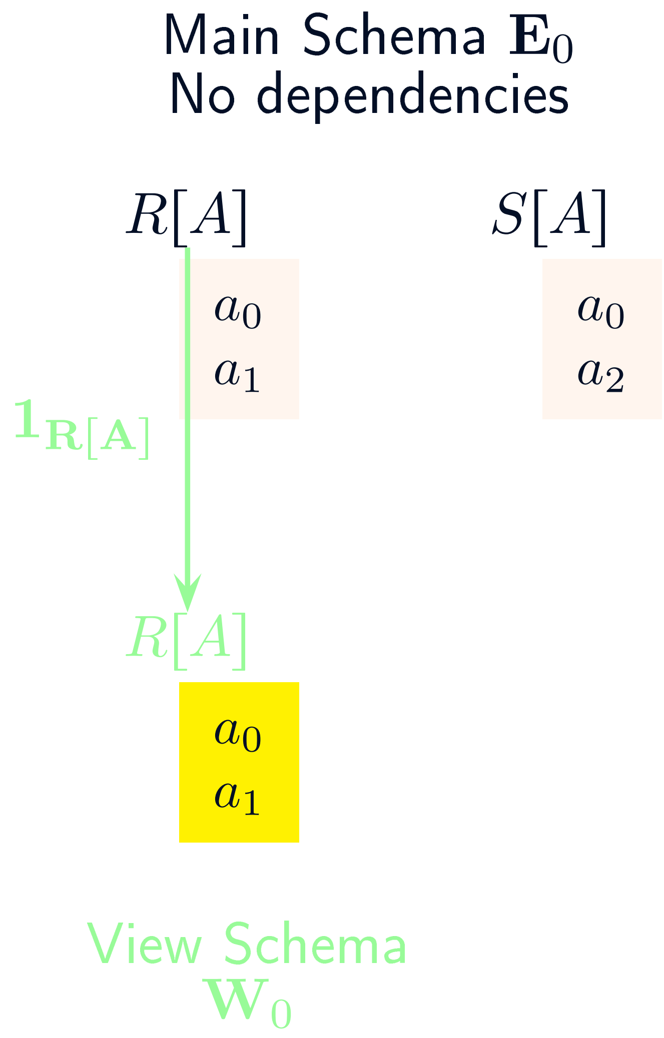

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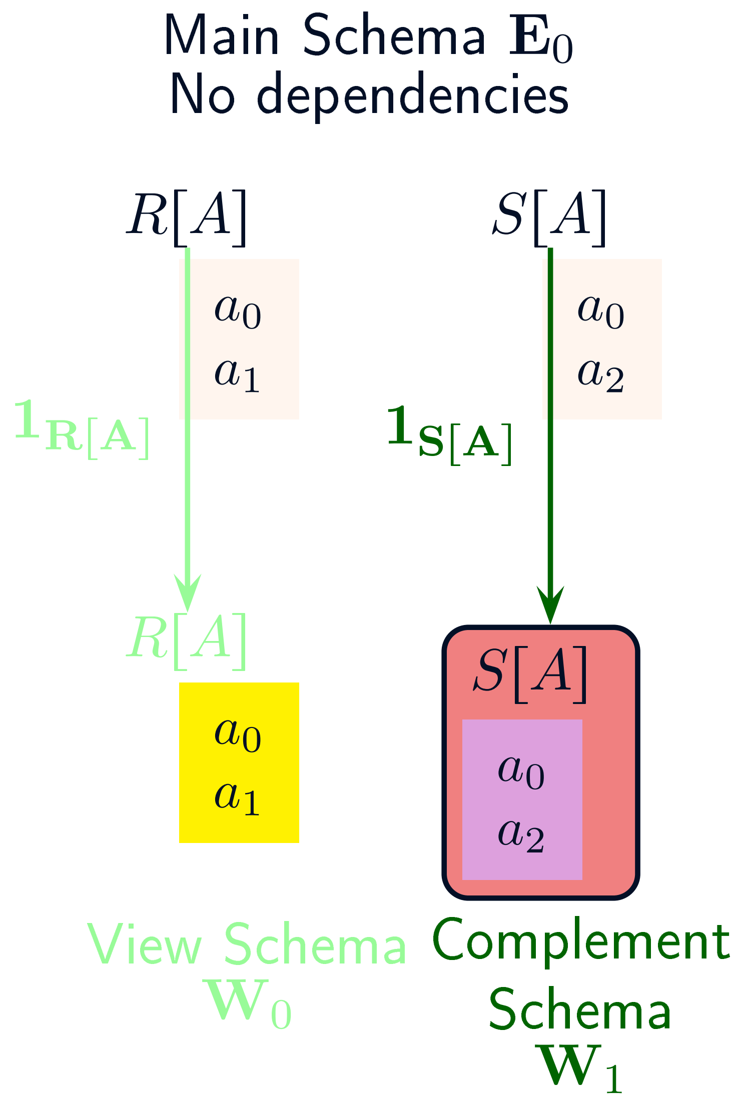

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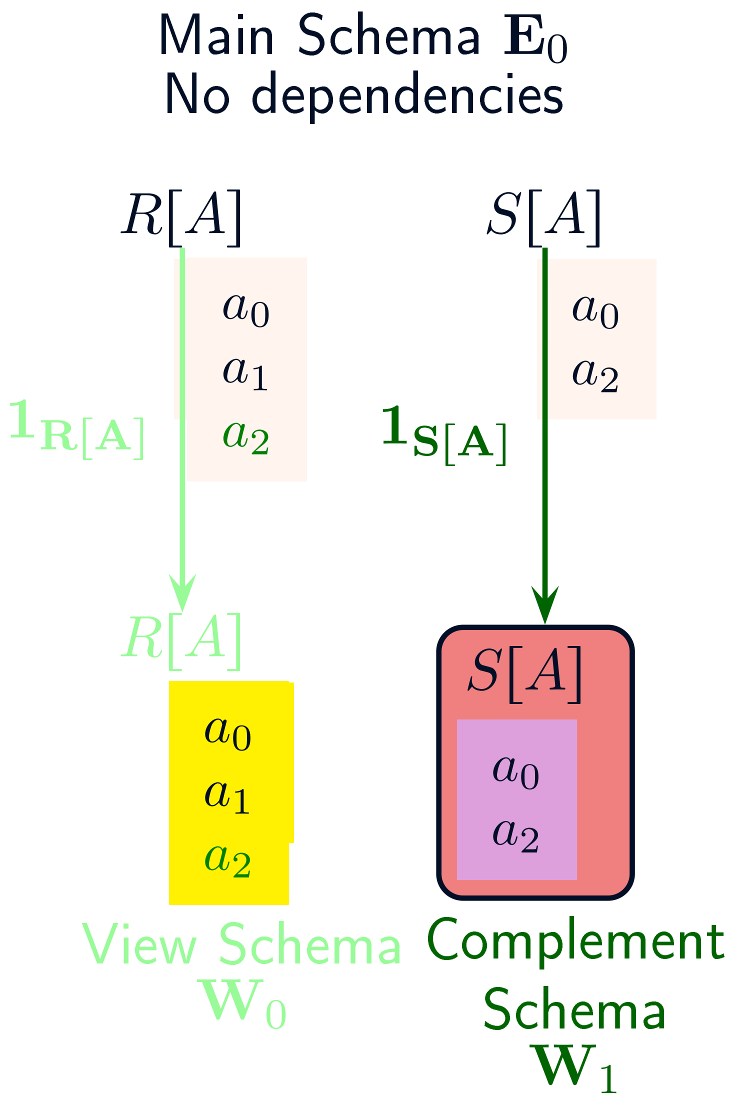

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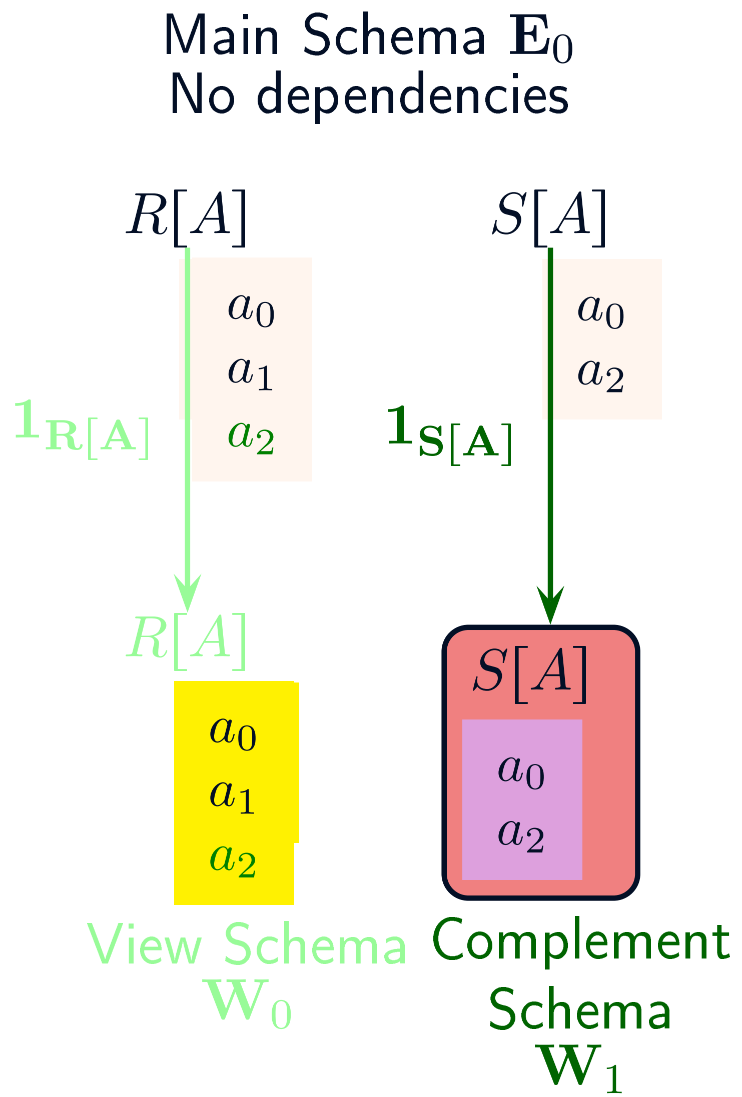

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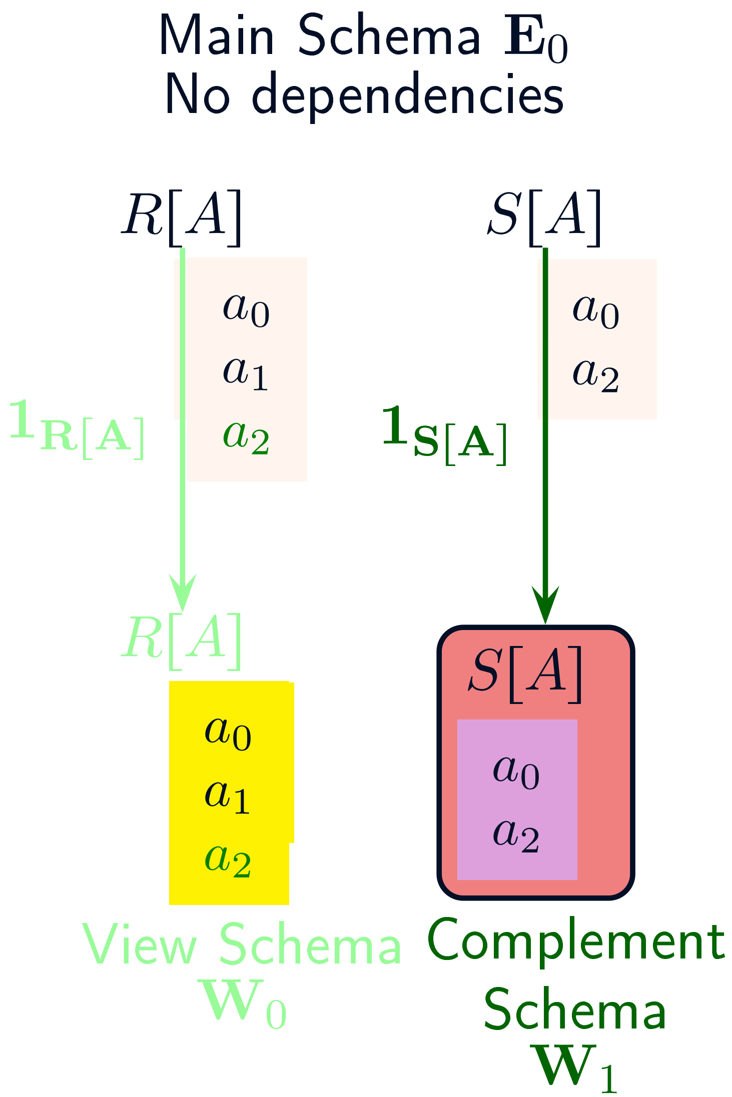

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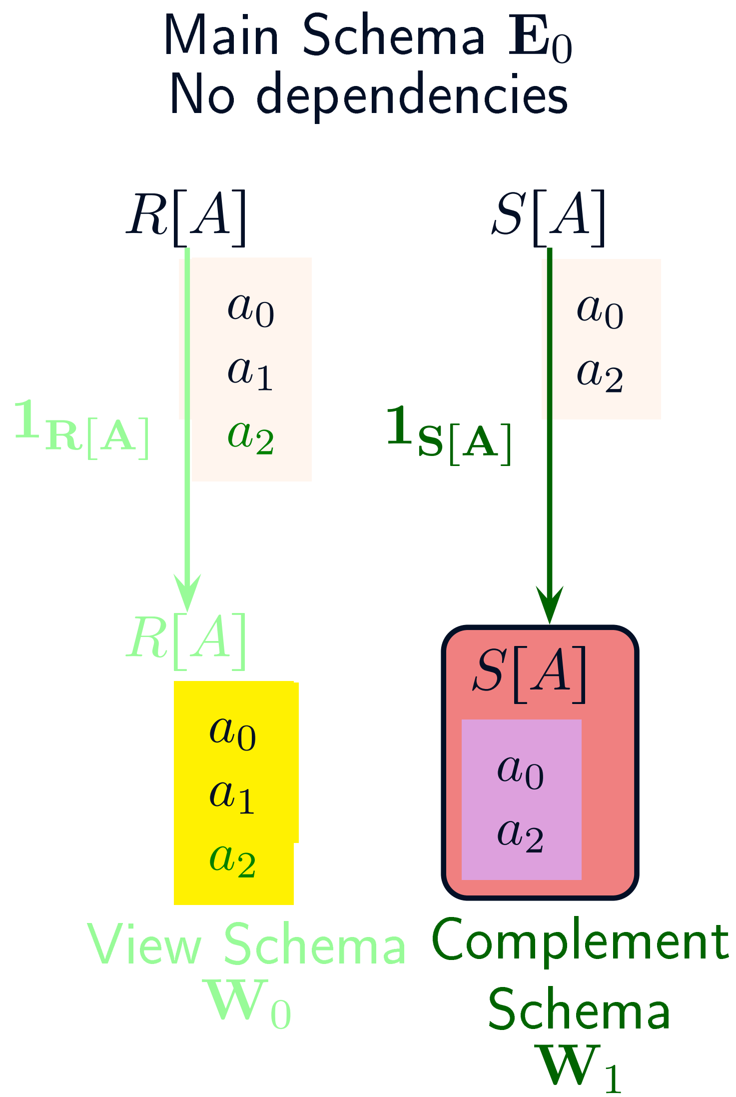
- Without further restrictions, complements are almost never unique.



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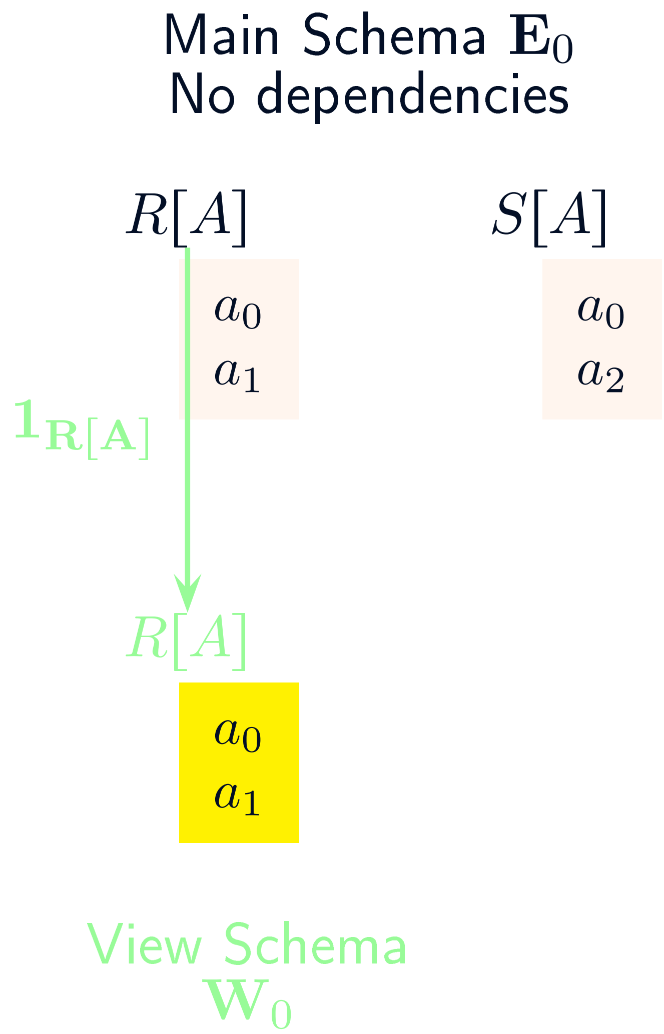

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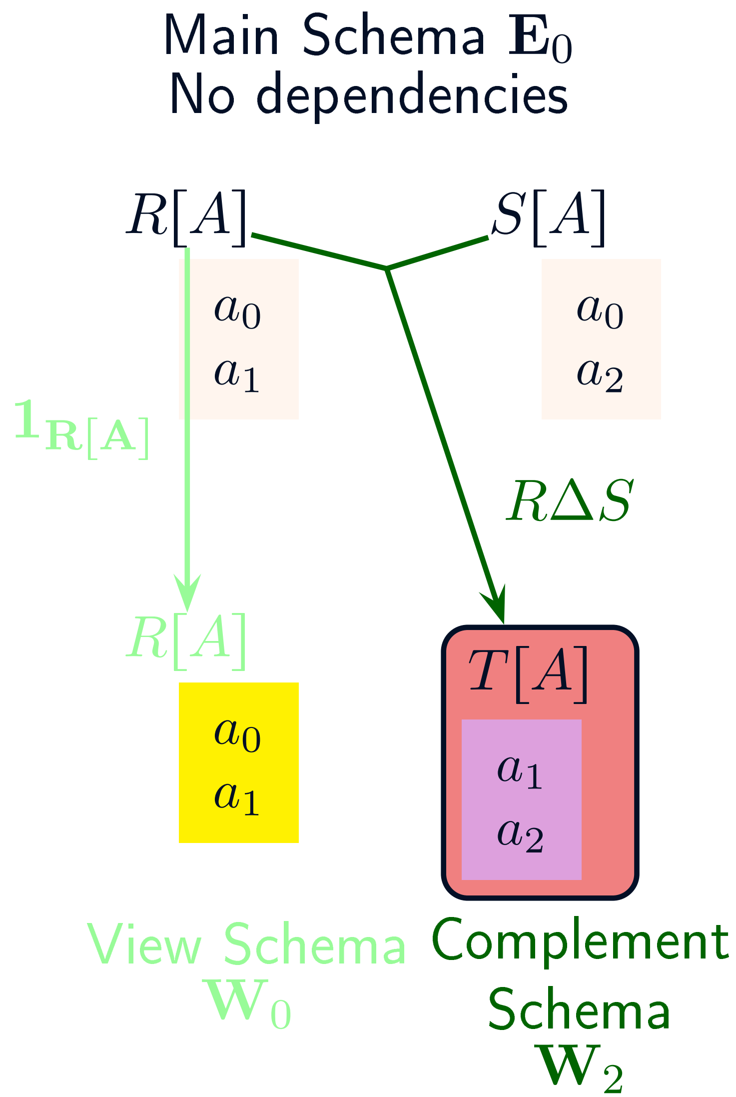

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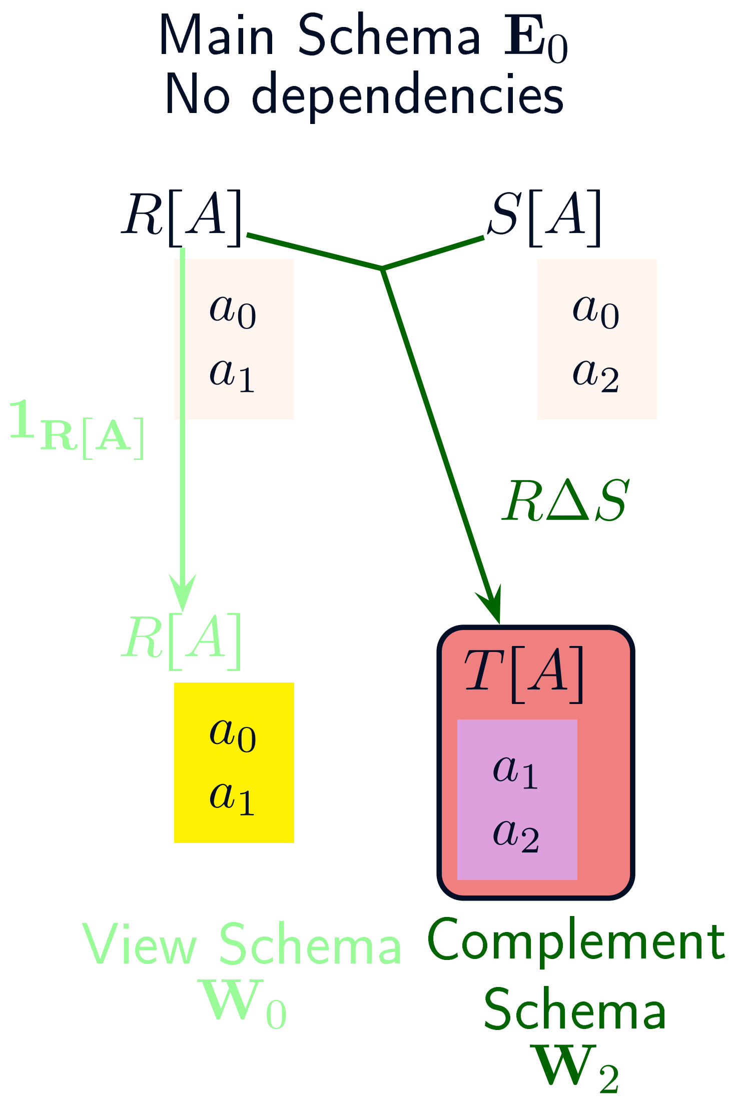

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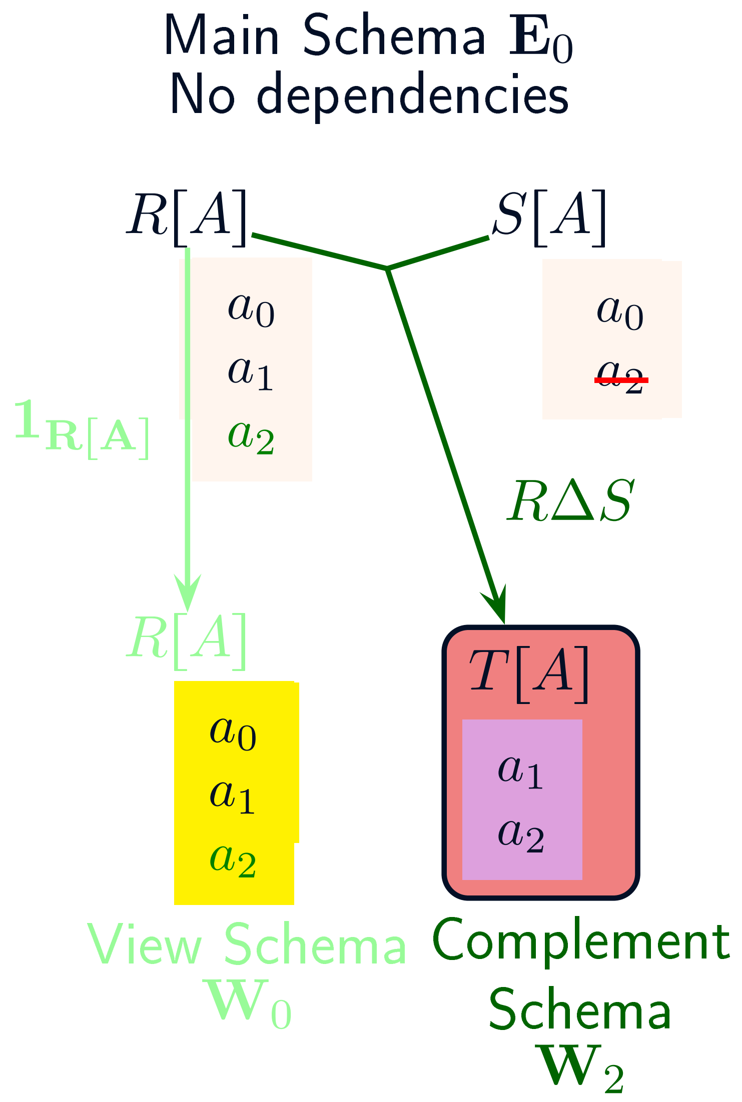

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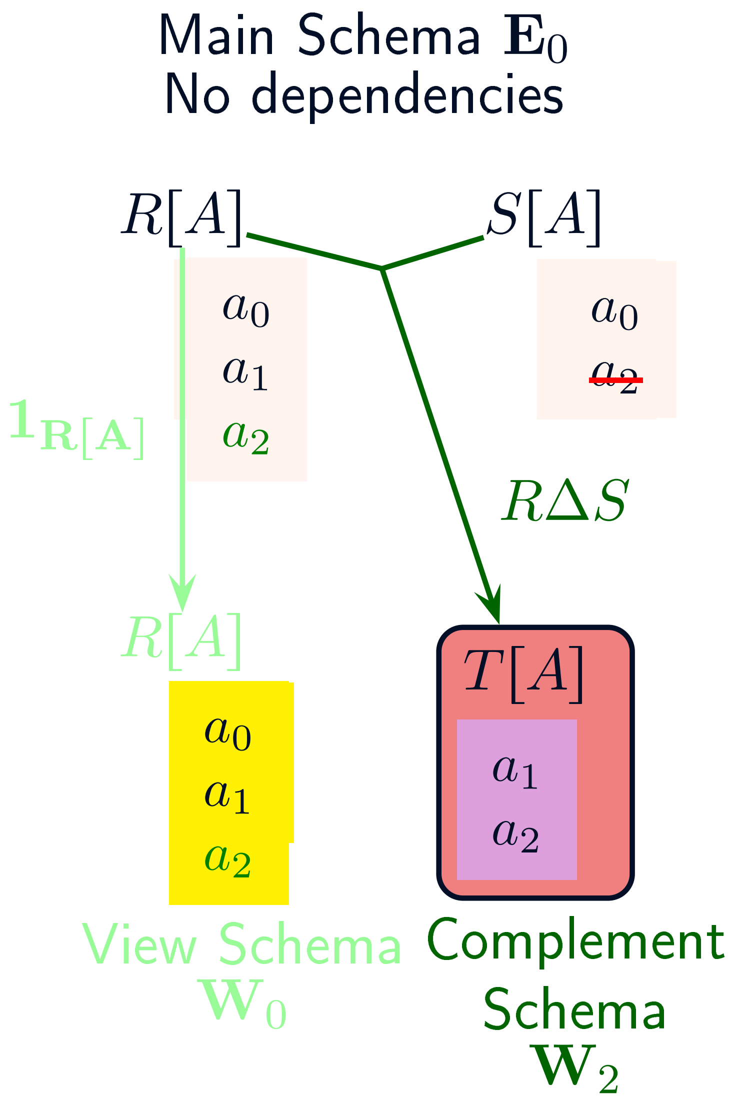
Question: How can these two complements be distinguished formally?



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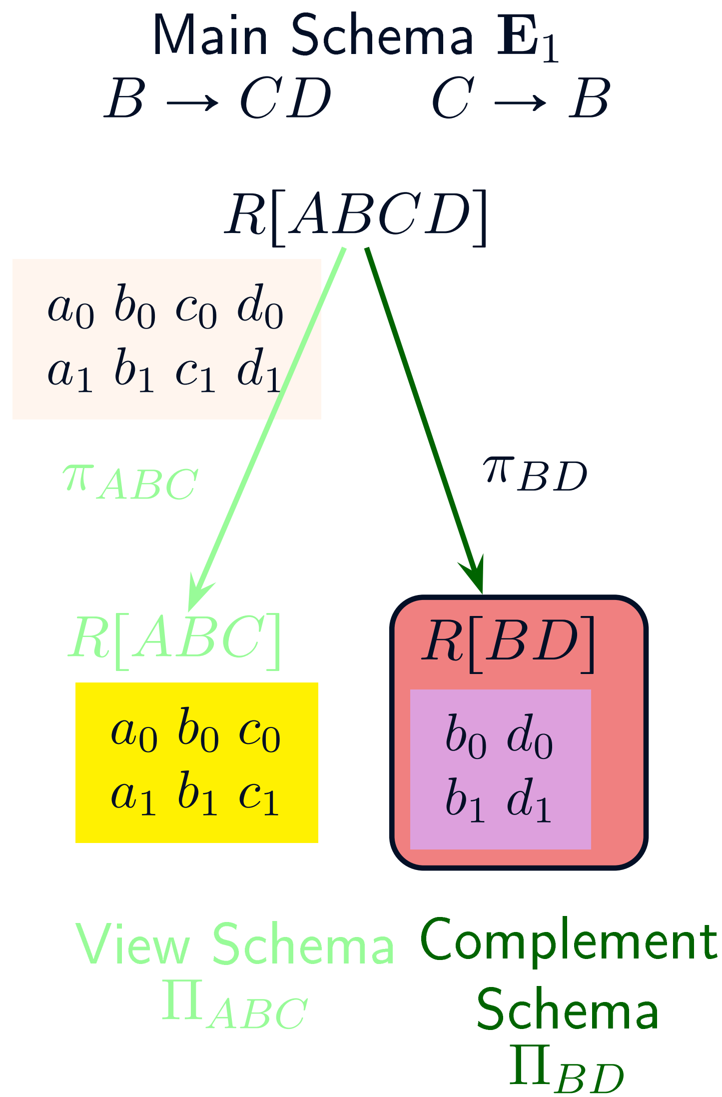
- It is useful to illustrate with a simple example.

- The view Π_{ABC} of the schema to the right has Π_{BD} as a natural monotonic meet complement.



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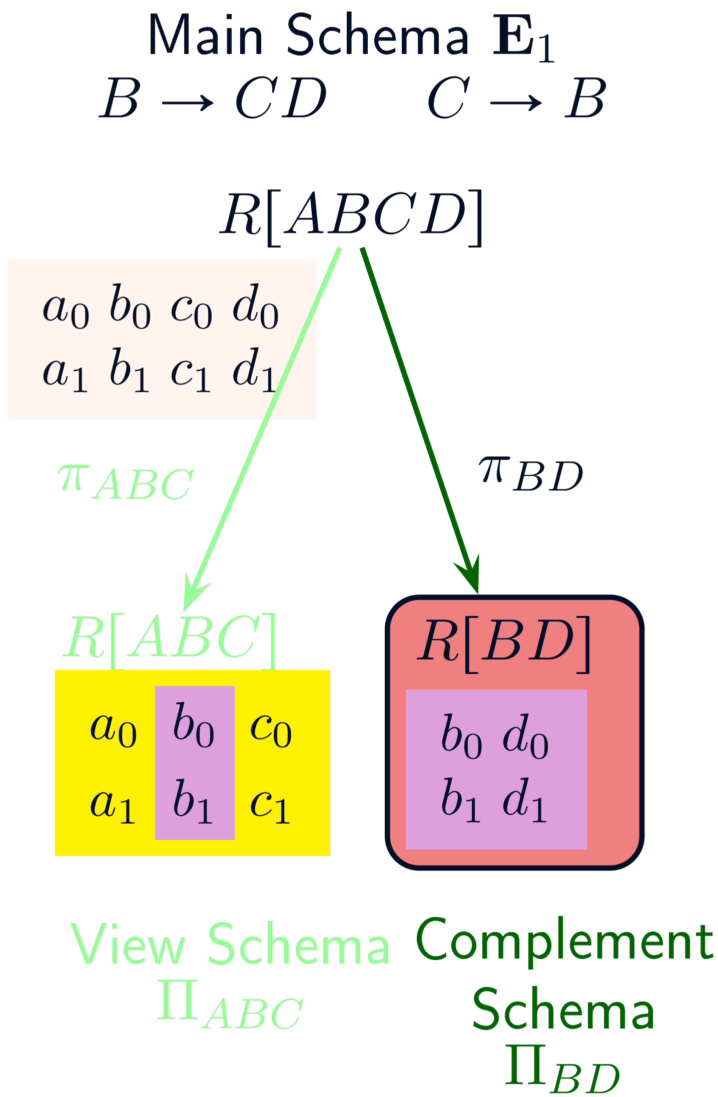

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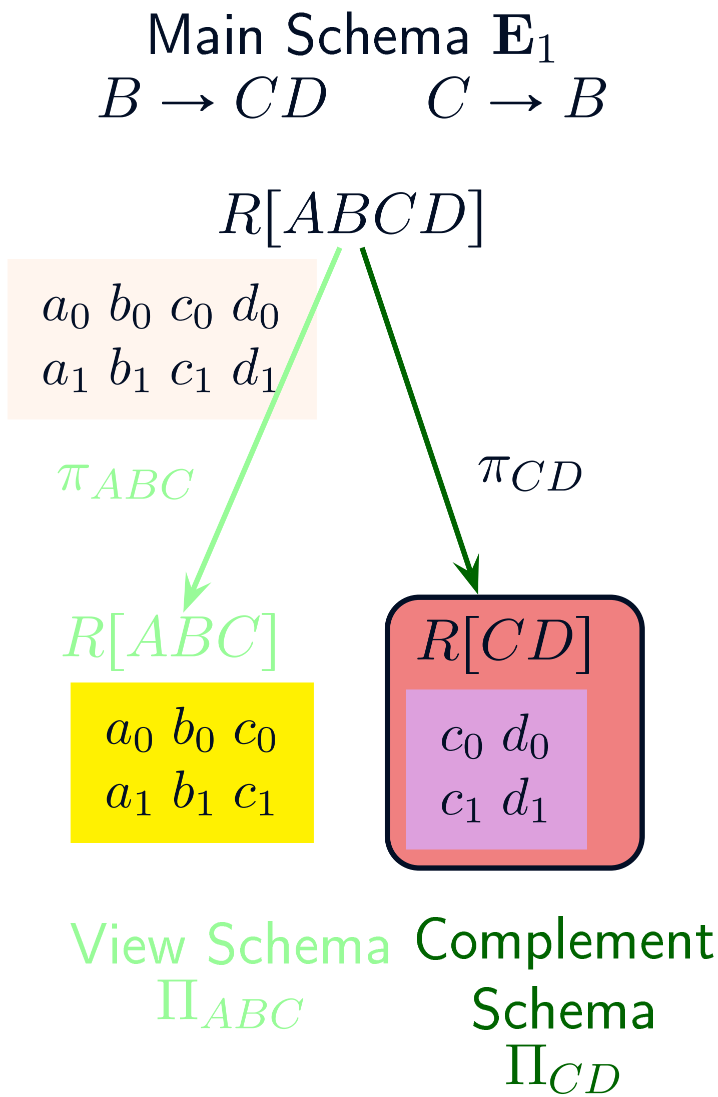

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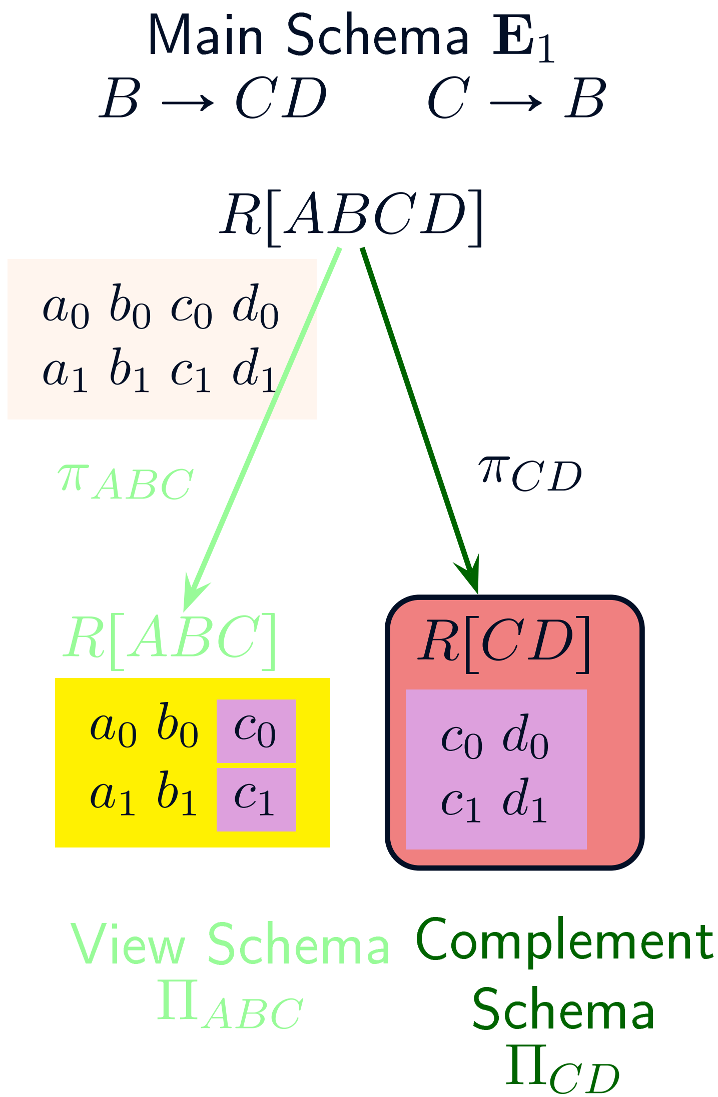

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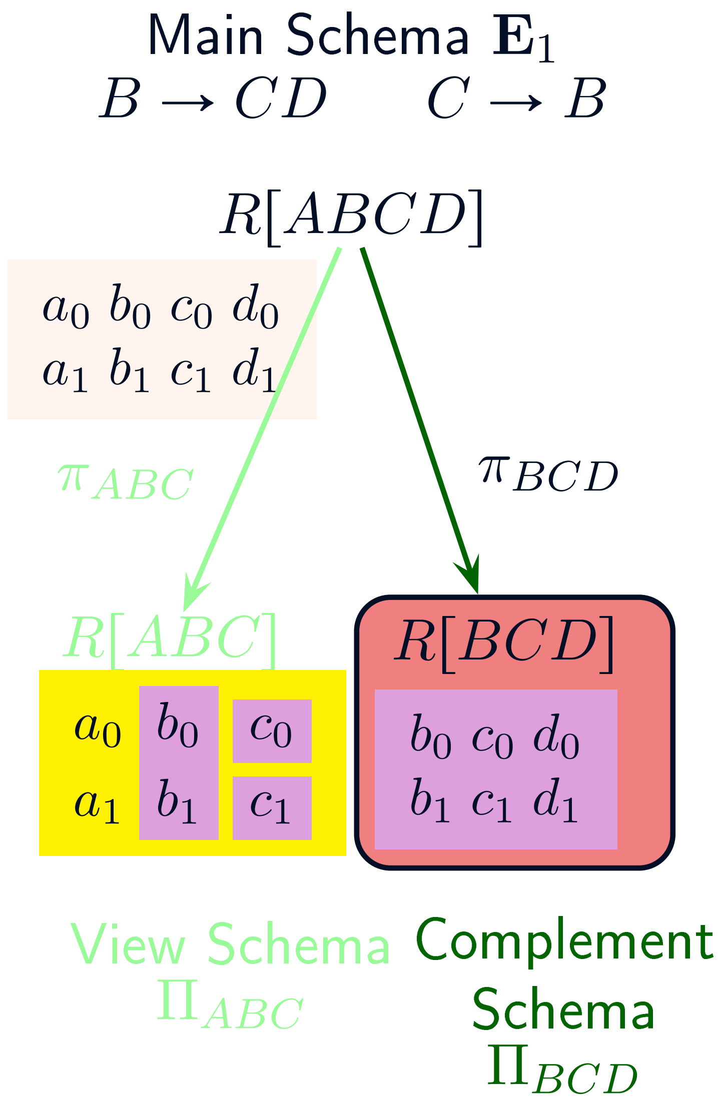

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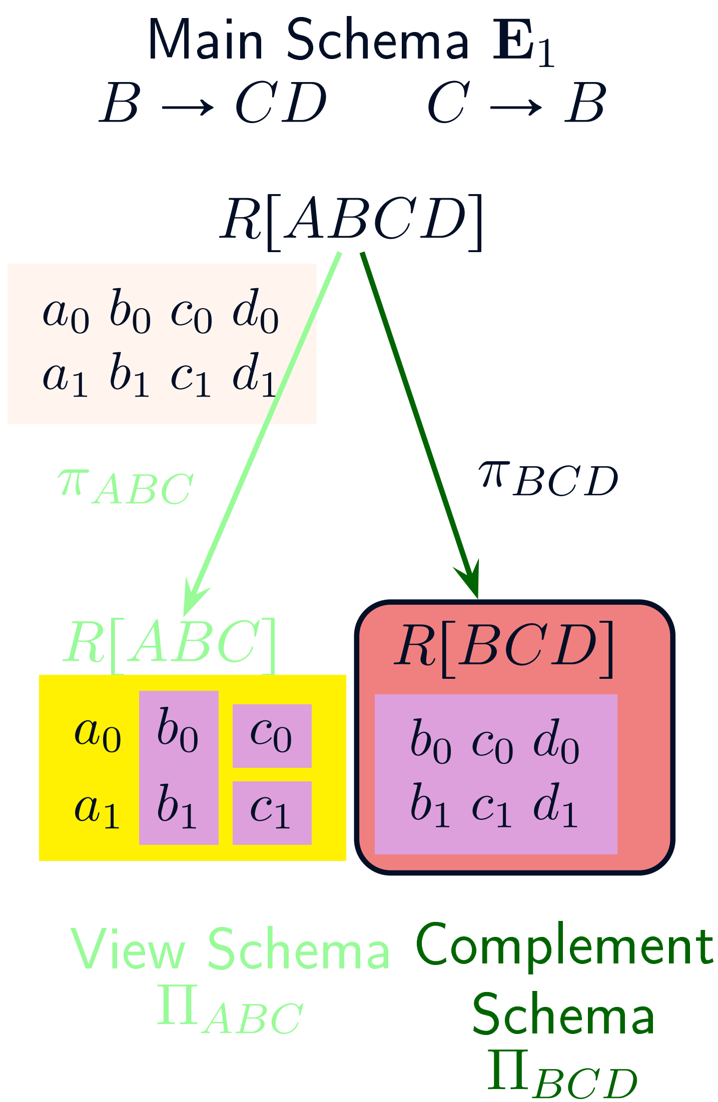

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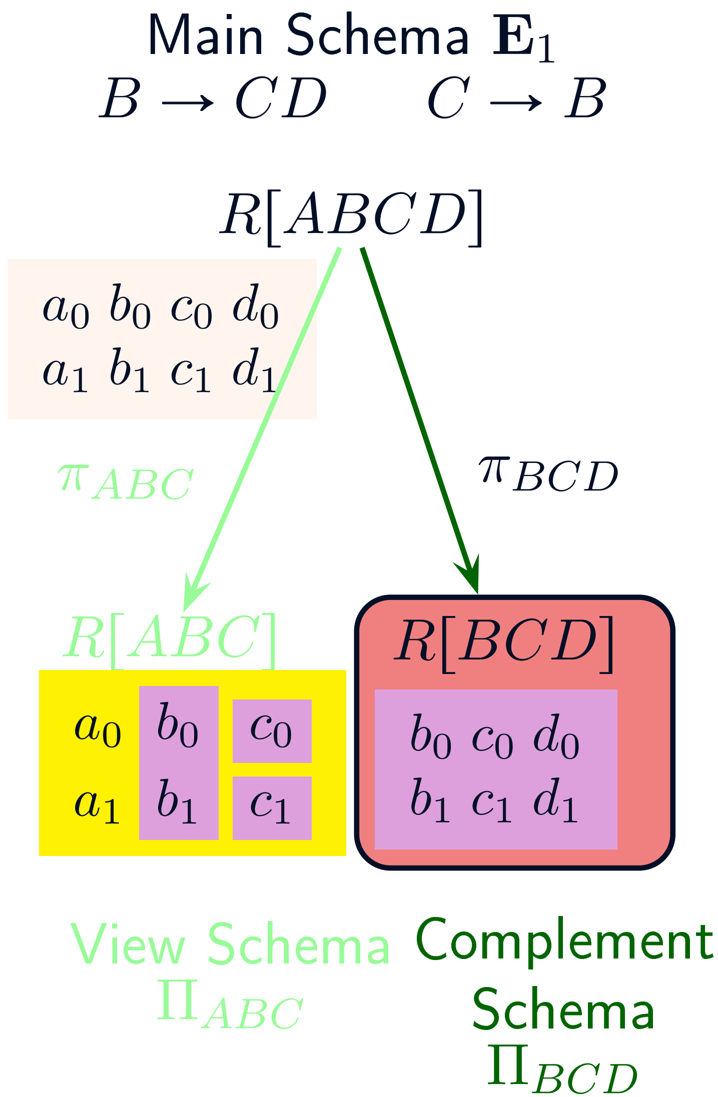
- The only updates allowable with both complements are those which hold Π_{BC} constant.
- The combined complement is effectively Π_{BCD}.
- There is no Π-complement which is more general than Π_{BD} or Π_{CD}.



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Example: For $(R[ABCD]$ with $\mathcal{F} = \{\bowtie [AB, BC]\}$, there is no (nontrivial) governing JD.

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- Consider the update (N, N') to Π_{CD} with $N = \{R(\mathrm{c}_1, \mathrm{d}_1), R(\mathrm{c}_2, \mathrm{d}_2)\}$. and $N' = \{R(\mathrm{c}_1, \mathrm{d}_2), R(\mathrm{c}_2, \mathrm{d}_1)\}$.

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- The view $\Pi_{ABC} \vee \Pi_{DE}$ which contains two projections, $R[ABC]$ and $R[DE]$, is a complement of Π_{CD}.
- Thus, (M, M') is a constant-complement reflection of (N, N') with complement $\Pi_{ABC} \vee \Pi_{DE}$.

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- Note that $\Pi_{ABC} \vee \Pi_{DE} \neq \Pi_{ABCDE}$, $\Pi_{CD} \vee \Pi_{EF} \neq \Pi_{CDEF}$; they are not even isomorphic.

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- A better definition of comparison: every LHS attribute set is a subset of a valid join of a RHS set.

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- For $\mathbf{W} \subseteq \mathbf{U}$, define $\text{JCompl}\langle \mathbf{W}, \mathcal{A}\rangle = \{\mathbf{U}_i \in \mathcal{A} \mid \mathbf{U}_i \nsubseteq \mathbf{W}\}$.

Theorem: $\bigvee\{\Pi_{\mathbf{U}_i} \mid \mathbf{U}_i \in \text{JCompl}\langle \mathbf{W}, \mathcal{A}\rangle\}$ is an optimal $\vee\Pi$-complement of $\Pi_{\mathbf{W}}$. □

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  Example: The tuple (b, c) for the view defined by $\pi_{AB}(R[ABC])$ corresponds to the Boolean query $(\exists z)(R(\mathrm{a}, \mathrm{b}, z))$.

  Example: The tuple $(\mathrm{a}, \mathrm{b}, \mathrm{c})$ for the view defined by $\sigma_{A=\mathrm{a}}(R[ABC])$ corresponds to the Boolean query $R(\mathrm{a}, \mathrm{b}, \mathrm{c})$.

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  Example: The two definitions $\pi_{BC}(\sigma_{A=\mathrm{a}_1}(R[ABC])$ and $\pi_{BC}(\sigma_{A=\mathrm{a}_2}(R[ABC])$; hide their selection constant in the sense that it is not visible in the view.

The Representation of $\sigma\exists\wedge+$-Views using sets of Boolean Queries

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Theorem; There is a natural correspondence between concrete and abstract views. □

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Example: For the schema $R[ABC]$ constrained by $\bowtie [AB, BC]$, the decomposition basis consists of elements of the form $(\exists z)(R(\mathrm{a}, \mathrm{b}, z))$ and $(\exists x)(R(x, \mathrm{b}, \mathrm{c}))$.

Optimal Complements in a General Setting

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- There are of course many details which have been omitted.

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- That theory provides a beginning to a more general theory but leaves several further directions.

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