FD Covers and Universal Complements of Simple Projections

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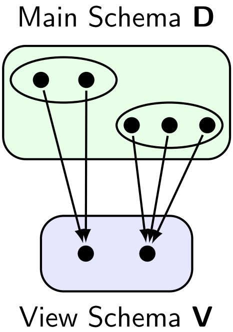

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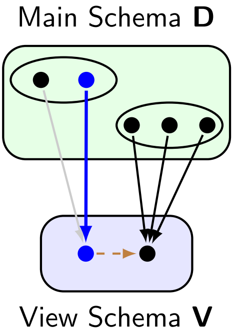

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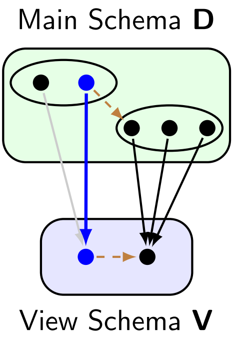

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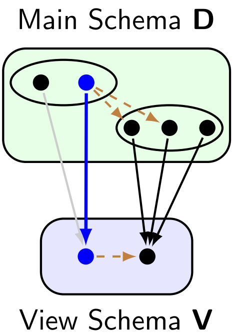

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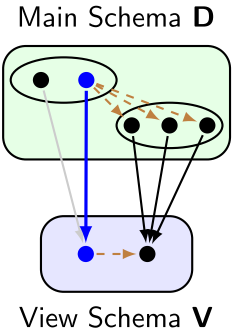

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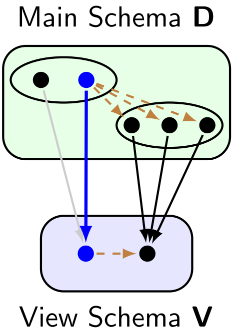

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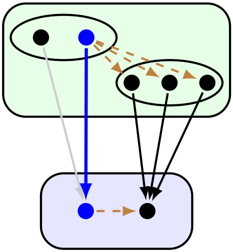

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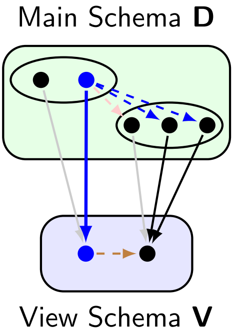

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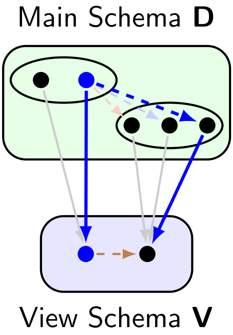

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- The research thus relates in particular to *closed* update strategies.

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Question: How is a closed update strategy realized?

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$R[ABC]$
$B \to C$

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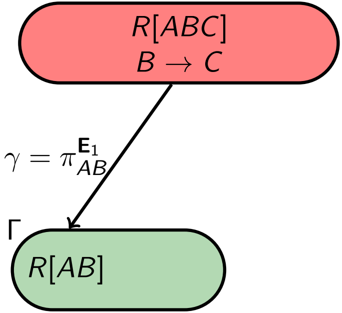

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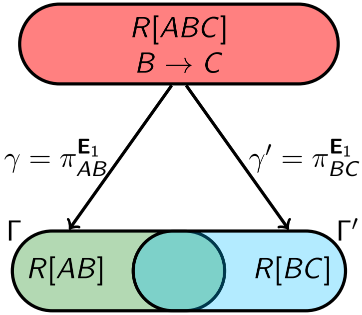

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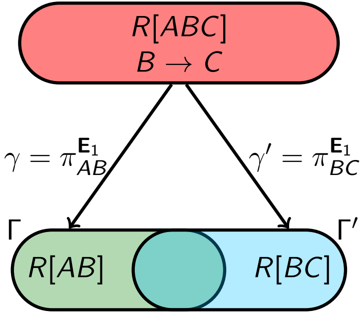

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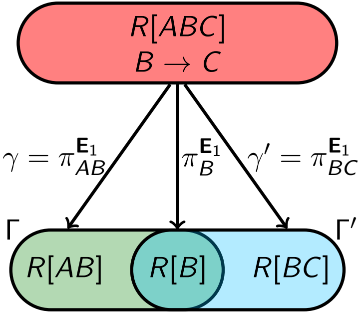

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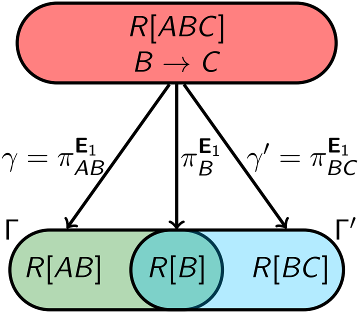

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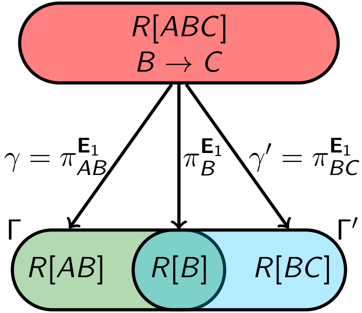

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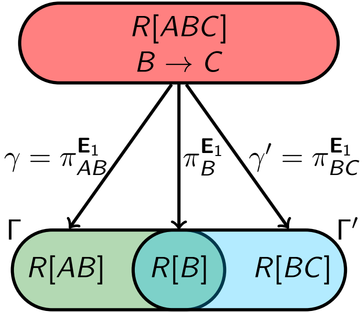

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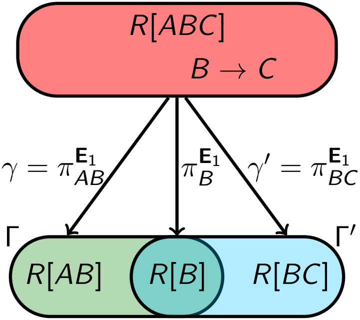

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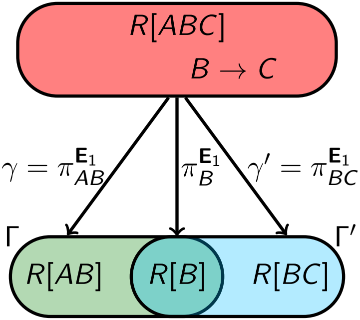

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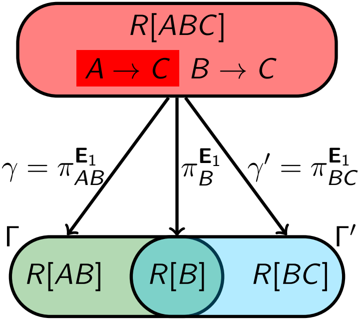

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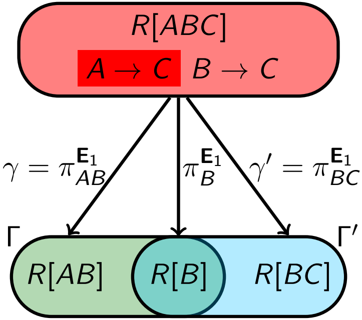

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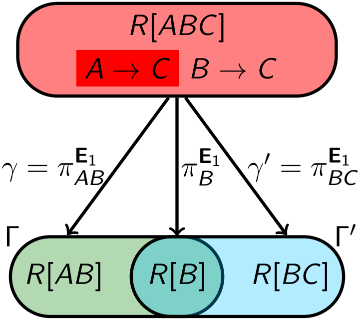

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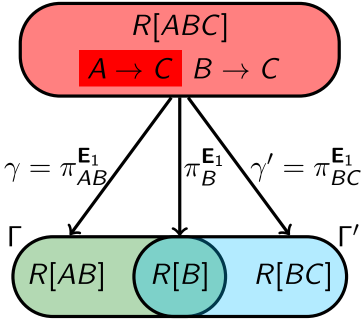

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Meet complement: A complement which induces admissibility invariance.

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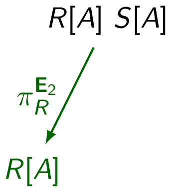

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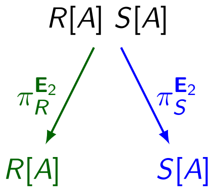

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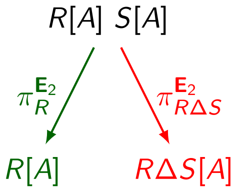

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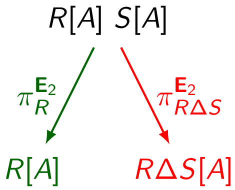

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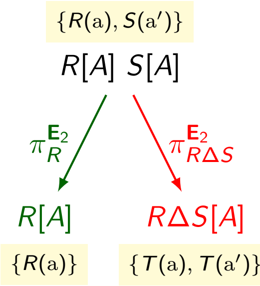

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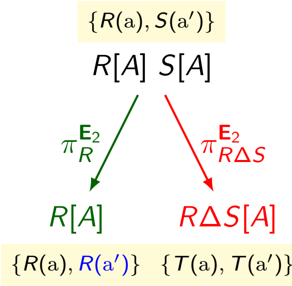

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View update: $u = (\{R(\mathrm{a})\}, \{R(\mathrm{a}), R(\mathrm{a}')\})$ on $\Pi_R^{\mathbf{E}_2}$. (Insert $R(\mathrm{a}')$).

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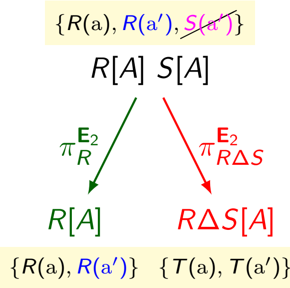

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- Another complement: $\Pi^{\mathbf{E}_2}_{R\Delta S} = (T[A], \pi^{\mathbf{E}_2}_{R\Delta S})$.
  - $T[x] \Leftrightarrow (R(x) \wedge (\neg S(x)) \vee ((\neg R(x)) \wedge S(x)))$.

Current state of main schema $\mathbf{E}_2$: $M_1 = \{R(\mathrm{a}), S(\mathrm{a}')\}$.

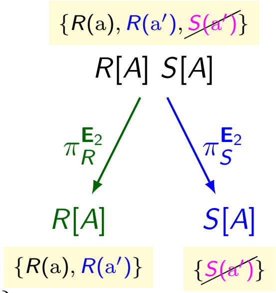
View update: $u = (\{R(\mathrm{a})\}, \{R(\mathrm{a}), R(\mathrm{a}')\})$ on $\Pi^{\mathbf{E}_2}_{R}$. (Insert $R(\mathrm{a}')$).

New state of $\mathbf{E}_2$: $M_2 = \{R(\mathrm{a}), R(\mathrm{a}')\}$ with constant complement $\Pi^{\mathbf{E}_2}_{R\Delta S}$.

The Achilles' Heel of Constant Complement

Achilles' Heel of the constant-complement strategy: Complements need not be unique.

- The reflection of a view update may depend upon the choice of complement.



Example: $\mathbf{E}_2$ has two relation symbols $R[A]$ and $S[A]$.

- The view to be updated is $\Pi_R^{\mathbf{E}_2}$.
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- Note that *admissibility invariance* is satisfied in each case.

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Simple example: $\Pi^{\mathbf{E}_2}_{R\Delta S}$ on the previous slide is ruled out since it is not order preserving.

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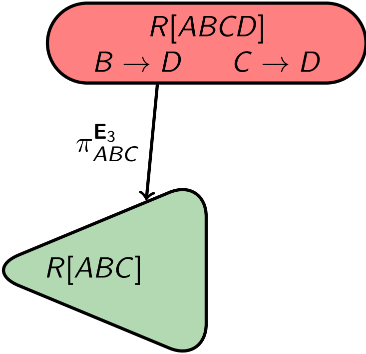

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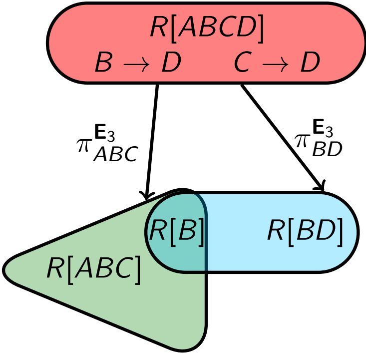

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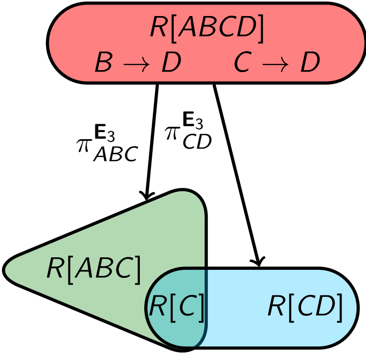

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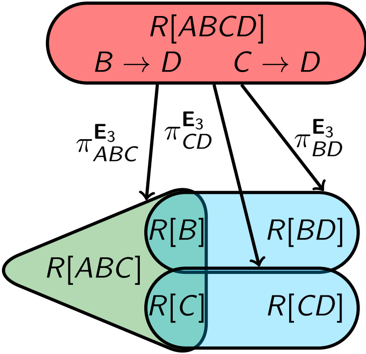

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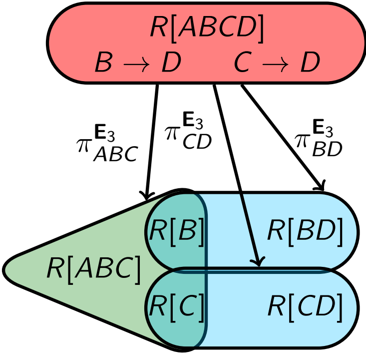

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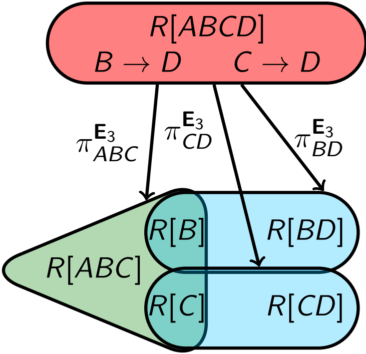

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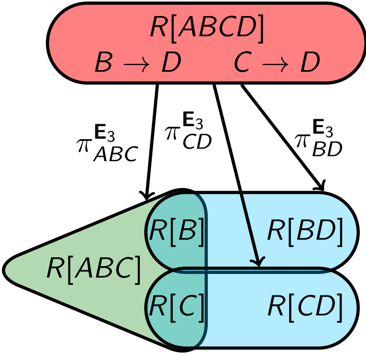

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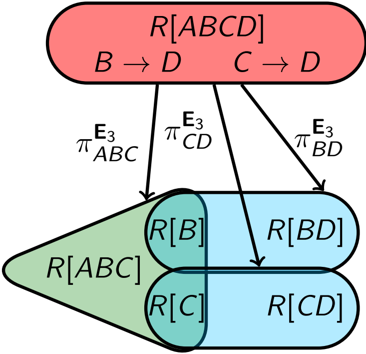

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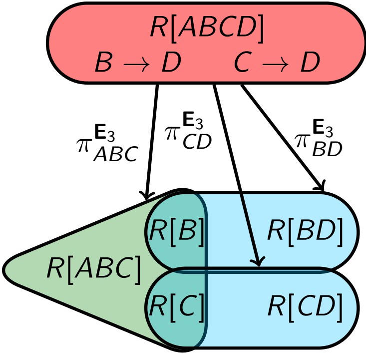

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- Admissibility-invariant examples are possible but a bit more complex.

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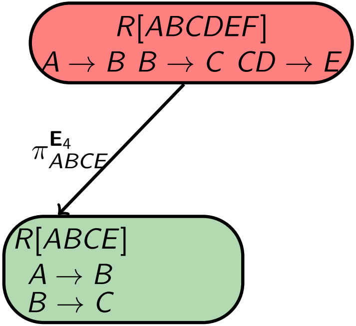

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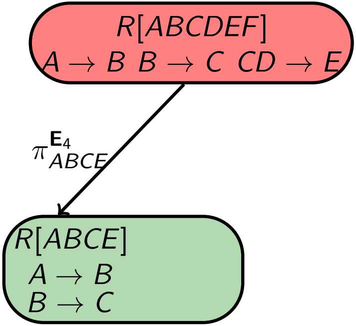

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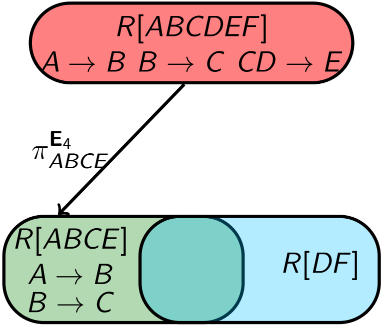

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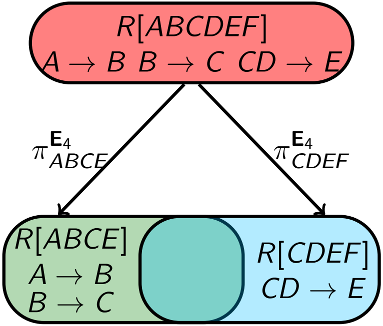

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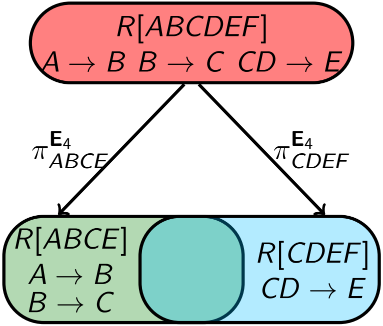

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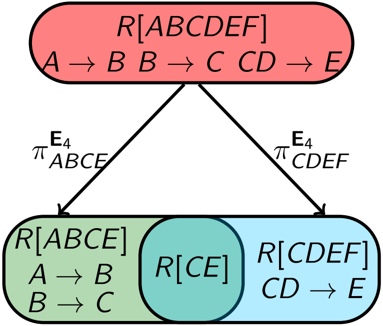

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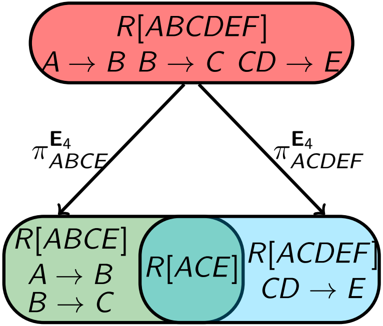

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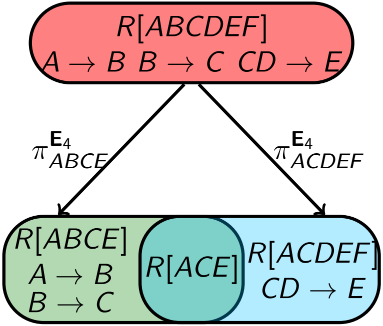

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Theorem: If $\mathcal{F}$ has the strong cover property, then, then every projection of **E** has a universal meet Π-complement. □

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Strong cover property: $\mathcal{F}$ has the *strong cover property* if the following two conditions are satisfied.

- $\mathcal{F}$ has a unique canonical cover.
- $\mathcal{F}$ has a unique key.

Theorem: If $\mathcal{F}$ has the strong cover property, then, then every projection of **E** has a universal meet Π-complement. □

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Short answer: There are unfortunately very simple examples with no universal meet complement.

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Example: Let $\mathbf{E}_6 = (R[ABCD], \mathcal{F}_6)$, with $\mathcal{F}_6 = \{AB \to C, C \to B, D \to A\}$.

- Although $\mathcal{F}_6$ is its own unique cover, it has two keys, BD and CD, and so $\Pi^{\mathbf{E}_6}_{ABC}$ has two minimal meet Π-complements, $\Pi^{\mathbf{E}_6}_{ABD}$ and $\Pi^{\mathbf{E}_6}_{ACD}$.

Extension to Simple Equivalences

- $B_1 \leftrightarrow B_2$ illustrates a *simple equivalence*, and B_1 and B_2 are *simply equivalent*.

$$R[AB_1B_2C]$$
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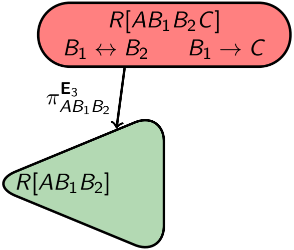
  - They are equivalent up to a renaming of simply equivalent attributes.



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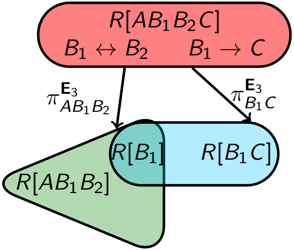

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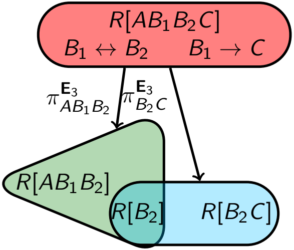

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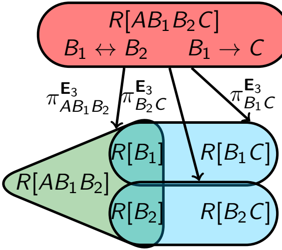

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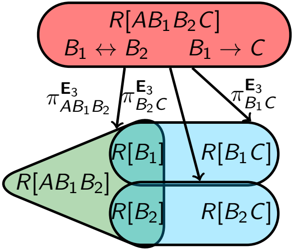

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Theorem: If $\mathcal{F}$ has the strong equivalence-cover property and is *free of complex triples* (a technical condition usually met in practice), then every Π-view of $\mathbf{E}$ admits quasi-universal Π-complements. □

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- Within the context of a more general model (such as view congruences for admissibility invariance and order structure for reflection invariance), identify the fundamental principles which underlie the existence of universal complements.
- Integrate the study of universal complements with the optimal interconnection of database components.