Implicit Representation of Bigranular Rules for Multigranular Data

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The Idea of Multigranular Attributes

Spatio-temporal attributes

Thematic attributes

cmn = comuna/county
prv = provincia/province
rgn = región/region

Place	Time	Births
Puerto_Montt_cmn	*Y2017Q1*	b_1
Llanquihue_prv	*Y2017Q1*	b_2
Los_Lagos_rgn	*Y2017Q1*	b_3
Puerto_Montt_cmn	*Y2017*	b_4

Granules: The domain values are called *granules*.

Granular order: The granules of spatial and temporal attributes have inherent order structure.

Spatial containment: *Puerto_Montt_cmn* $\sqsubseteq$ *Llanquihue_prv* $\sqsubseteq$ *Los_Lagos_rgn*

Temporal interval containment: *Y2017Q1* $\sqsubseteq$ *Y2017*

Typical constraints: Functional dependency (FD) $\{\text{Place}, \text{Time}\} \rightarrow \text{Births}$.

- The number of births is monotonic w.r.t. space and time, so $b_1 \leq b_2 \leq b_3$, $b_2 \leq b_4$.

Lattice-Like Operations on Granules

Place	Time	Births
Osorno_prv	*Y2017Q1*	b_1
Llanquihue_prv	*Y2017Q1*	b_2
Chiloé_prv	*Y2017Q1*	b_3
Palena_prv	*Y2017Q1*	b_4
Los_Lagos_rgn	*Y2017Q1*	b_5

Join: The four provinces join to the region.

$$Los\_Lagos\_rgn = \bigsqcup\{Osorno\_prv, Llanquihue\_prv, Chiloé\_prv, Palena\_prv\} \quad .$$

Meet: Distinct provinces are disjoint (six possibilities in all).

$$\bigsqcap\{Osorno\_prv, Llanquihue\_prv\} = \bot$$

Disjoint Join: The four provinces join *disjointly* to the region.

$$Los\_Lagos\_rgn = \bigsqcup^{\perp}\{Osorno\_prv, Llanquihue\_prv, Chiloé\_prv, Palena\_prv\} \quad .$$

Consequence: $\sum_{i=1}^{4} b_i = b_5$.

Observation: These lattice-like operations are *partial*.

Approaches to Modelling Multigranular Domains

Question: How can multigranular attributes be modelled for effective implementation? Two possibilities:

Single-structure model: Widely used in *Geographic Information Systems*.

Fix a domain $\mathcal{D}$; semantics of granule g defined by a subset $\mathrm{Sem}(g) \subseteq \mathcal{D}$.

- For a spatial attribute, $\mathcal{D} =$ suitable subset of $\mathbb{R} \times \mathbb{R}$.
- $\mathrm{Sem}(g)$ the geographic region represented by g.

Advantages: Extensive model; well-developed theory and practice.

Disadvantages: Extreme resource demands, both space and time.

Constraint-based model: Work directly with constraints of the form $g_1 \sqsubseteq g_2$, $g \sqsubseteq \biguplus\{g_i \mid 1 \leq i \leq k\}$, and $g = \biguplus\{g_i \mid 1 \leq i \leq k\}$, among others.

Advantages: Only represent as much information as needed.

Challenges: Constraint inference,

Constraint retrieval (based upon features), Constraint consistency.

A Logic for Multigranular Domains

- A logic for representing knowledge within multigranular attributes has been developed [Hegner & Rodríguez, 2016, 2017].

Granule Expressions (terms): $g, \quad \bigsqcup\{g \mid g \in S\}, \quad \bigsqcap\{g \mid g \in S\}, \quad \bot, \quad \top$

Granule Rules (sentences): Combine granule expressions using $\sqsubseteq$ (and equality). Examples: $g_1 \sqsubseteq g_2, \quad \bigsqcap\{g_1, g_2\} = \bot$.

Semantics/Models: Set semantics. Fix a domain $\mathcal{D}$.

- A model assigns to each granule a semantics $\mathsf{Sem}(g) \subseteq \mathcal{D}$ in a manner which respects the operations.

Example: $g_1 \sqsubseteq g_2$ iff $\mathsf{Sem}(g_1) \subseteq \mathsf{Sem}(g_2)$.

Satisfiability (of a set of rules): Related to distributivity of order operators.

- Very complex problem in theory.
- Not a problem in practice — axioms are models of "real" things.

Needs for an inference/lookup mechanism: The theory does not provide any such mechanism, beyond raw testing.

Goal of this work: Provide such a mechanism for the *common case*.

Requirements for Join-Rule Lookup

Join rule: One of the following forms: $g = \bigsqcup S \quad g = \biguplus S$

Head $g \sqsubseteq \bigsqcup S \quad g \sqsubseteq \biguplus S$ Body

Primary Lookup requirements:

Head lookup: Given a granule g', find all rules with head g'.

Body lookup: Given a set T of granules, find all rules with T contained in the body ($T \subseteq S$).

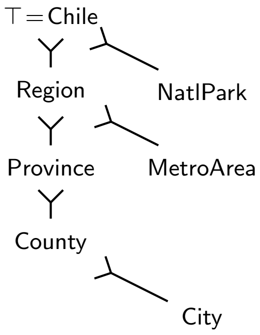
Complication: In the general case, the rules to be found may need to be derived first.

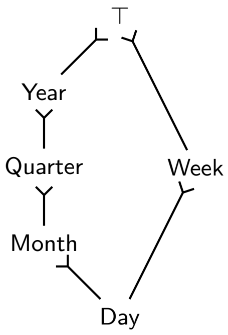
- A *knowledge base* of rules, not just a *database*.

Goal: For rule retrieval, *make the common case fast*.

Question: What is the common case?

Granularities — Organizing Granules





- The granules of each attribute are partitioned into a hierarchy of *granularities*.

Order: $G_1 \leq G_2 \Leftrightarrow ((\forall g_1 \in \text{Granules}\langle G_1 \rangle)(\exists g_2 \in \text{Granules}\langle G_2 \rangle)(g_1 \sqsubseteq g_2))$.

Examples: Every county is contained in a (unique) province.
Every day is contained in a (unique) week.

Disjointness: Distinct granules of the same granularity are disjoint.

Common Cases of Bigranular Rules

A common case: A family of *bigranular* rules between a pair of granules.

Bigranular rule: All granules in the body are of the same granularity.

- The head is necessarily of a different granularity.

Common case 1: Equality-join order property:

$G_1 \trianglelefteq G_2 \stackrel{\text{def}}{=}$ every granule of G_2 is the (disjoint) join of granules of G_1.

Example of common case 1: Province $\trianglelefteq$ Region.

$Los\_Lagos\_rgn = \bigsqcup^{\perp} \{Osorno\_prv, Llanquihue\_prv, Chiloé\_prv, Palena\_prv\}$.

$BíoBío\_rgn = \bigsqcup^{\perp} \{Arauco\_prv, BíoBío\_prv, Concepción\_prv, \cancel{Ñuble\_prv}\}$.

Common case 2: Subsumption-join order property:

$G_1 \lessdot G_2 \stackrel{\text{def}}{=}$ every granule of G_2 is contained in the (disjoint) join of granules of G_1.

Example of common case 2: MetroArea $\lessdot$ Province

$Gran\_Puerto\_Montt\_urb \sqsubseteq \bigsqcup^{\perp} \{Puerto\_Montt\_cmn, Puerto\_Varas\_cmn\}$

Resolvability and the Nondisjointness Relation

Context: Let $\mathcal{C}$ denote the set of constraints which hold on the multigranular attribute under consideration.

- Given a rule φ, there are three possible cases.

  (a) $\mathcal{C} \models \varphi$ (b) $\mathcal{C} \models \neg\varphi$ (c) Neither of these

Resolvability: The rule φ is *resolvable* from $\mathcal{C}$ if one of (a) or (b) holds.

- Written $\mathcal{C} \models^{\pm} \varphi$.

Context: $\langle G_1, G_2 \rangle$ a pair of granularities.

Full disjointness resolvability: $\langle G_1, G_2 \rangle$ is *fully disjointness resolvable* if $(\forall g_1 \in \mathrm{Granules}\langle G_1 \rangle)(\forall g_2 \in \mathrm{Granules}\langle G_2 \rangle)(\mathcal{C} \models^{\pm} (\prod\{g_1, g_2\} = \bot))$.

A compact relational representation under full disjointness resolvability:

$$\mathrm{NRel}_{\langle G_1, G_2 \rangle} = \{\langle g_1, g_2 \rangle \mid \mathcal{C} \models (\prod\{g_1, g_2\} \neq \bot)\}$$

- $\mathcal{C} \models ((\prod\{g_1, g_2\} = \bot))$ iff $(g_1, g_2) \notin \mathrm{NRel}_{\langle G_1, G_2 \rangle}$.

Symmetry: Note that $\mathrm{NRel}_{\langle G_1, G_2 \rangle} = \mathrm{NRel}_{\langle G_2, G_1 \rangle}$ always holds.

- Use $\mathrm{NRel}_{\langle G_1, G_2 \rangle}$ rather than $\mathrm{DRel}_{\langle G_1, G_2 \rangle}$ (the corresponding relation for disjointness) because $\mathrm{NRel}_{\langle G_1, G_2 \rangle}$ is usually much smaller.

Main Representation Theorem for $\trianglelefteq$

Recall: $G_1 \trianglelefteq G_2$ iff

$$(\forall g_2 \in \mathsf{Granules}\langle G_2 \rangle)(\exists S \subseteq_f \mathsf{Granules}\langle G_1 \rangle)(\mathcal{C} \models (g_2 = \bigsqcup S)).$$

Main Theorem: If $G_1 \trianglelefteq G_2$ holds, then so do the following.

(a) $\langle G_1, G_2 \rangle$ is fully disjointness resolvable. In other words, for $G_1 \trianglelefteq G_2$ to hold, there must be complete information about disjointness of the collective granules of G_1 and G_2.

(b) In the above "recall" formula, for each $g_2 \in \mathsf{Granules}\langle G_2 \rangle$,

$$S = \{g_1 \in \mathsf{Granules}\langle G_1 \rangle \mid \langle g_1, g_2 \rangle \in \mathsf{NRel}_{\langle G_1, G_2 \rangle}\}.$$

In words, each $g_1 \in \mathsf{Granules}\langle G_2 \rangle$ is the join of those granules in $\mathsf{Granules}\langle G_1 \rangle$ with which it is not disjoint. This is the only possibility.

Fast head-driven lookup: To identify the body of the rule with head $g_2 \in \mathsf{Granules}\langle G_2 \rangle$, it suffices to find all matches to g_2 in $\mathsf{NRel}_{\langle G_1, G_2 \rangle}$.

Fast body-driven lookup: To identify the head of the rule with $S' \subseteq \mathsf{Granules}\langle G_1 \rangle$ as part of its body, it suffices to find the unique g_1 which matches every member of S' in $\mathsf{NRel}_{\langle G_1, G_2 \rangle}$.

Main Representation Theorem for $\oslash$

Recall: $G_1 \oslash G_2$ iff

$$(\forall g_2 \in \mathsf{Granules}\langle G_2\rangle)(\exists S \subseteq_f \mathsf{Granules}\langle G_1\rangle)(\mathcal{C} \models (g_2 \sqsubseteq \bigsqcup\!\!\!\!\perp S)).$$

- The result is similar to that of $\trianglelefteq$, with one additional condition necessary.
- With subsumption, a *resolved minimality* condition is necessary.

Motivating example: Consider MetroArea $\trianglelefteq$ Province.

Trivial "solution": *Gran_Puerto_Montt_urb* $\sqsubseteq$ Granules$\langle$Province$\rangle$.

Resolved minimality: In the "recall" formula, for any proper subset $S' \subsetneq S$,

$$(\mathcal{C} \models \neg(g_2 \sqsubseteq \bigsqcup\!\!\!\!\perp S')).$$

- In other words, if any element is removed from S, the assertion becomes false (not just fails to be true).
- Under those conditions, a theorem analogous to that for $\trianglelefteq$ holds.

Conclusions and Current Directions

Conclusions:

Representation of common-case join rules:

- Handles both equality- and subsumption-join order.
- Applies in a constraint-based framework with incomplete information.
- Uses only (non)disjointness information about granules.

Current Directions:

Implementation in MGDB: *MGDB* is a PostgreSQL-based multigranular DBMS under development at the University of Concepción.

- Test data of administrative and political subdivisions of Chile provide many instances of equality- and subsumption-join order.
- The ideas of this paper are being applied to the efficient implementation of the associated rules.