Canonical View Update Support through Boolean Algebras of Components

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ABSTRACT

The problem of reflecting updates from a view schema to the base schema in relational database systems is investigated. The basic strategy which is employed is that of translation relative to a constant complement, as developed by Bancilhon and Spyratos. We deal in particular with one of the apparent drawbacks of the constant complement approach, namely, the nonuniqueness of such complements. Our approach is to use only a specific set of well-behaved views, termed the *components* of the schema, as complements. The components form a Boolean algebra, and update reflection to the base schema is independent of choice of complement, as long as the complement is a component. We argue that such reflections of updates are the "natural" ones which should be allowed.

0. INTRODUCTION

0.1 Basic Problem Statement

Avoiding technical details for the moment, we think of a *relational database schema* **V** as a pair (Rel(**V**),Con(**V**)), where Rel(**V**) is a finite set of *relation symbols*, and Con(**V**) is a set of *integrity constraints*. A legal database for **V** is just an indexed set of relations, one for each $R \in$ Rel(**V**), which collectively satisfy the integrity constraints of Con(**V**). Denote the collection of all legal databases of **V** by LDB(**V**).

0.1.1 DEFINITION An *update specification for* **V** is a pair (s_1,s_2), where s_1 and s_2 are each elements of LDB(**V**). Think of s_1 as the current state of the schema, and s_2 as the desired next state. *Insertions*, *deletions*, and *replacements* are commonly considered special cases of this general notion of update.

In a *relational database system*, we assume that there is a main relational schema **D**, designated the *base schema*. This main schema encompasses the entire "enterprise" which is modelled, in the sense of the conceptual schema of the ANSI/SPARC proposal [ANSI75]. It is further assumed that any update specification (s_1,s_2) on this schema is supported (at least in principle), although certain special privileges may be required to effect such an update. In short, for the purposes of this paper, we consider the update problem on the base schema to be solved.

For reasons of security and ease of use, a user is not generally given direct access to the base schema. Rather, his window on the system is via a

view. A view is given by a pair $\Gamma = (\mathbf{V},\gamma)$, where $\mathbf{V}$ is a relational database schema, and $\gamma : \mathbf{D} \rightarrow \mathbf{V}$ is a *database mapping*. Such a mapping defines a function $\gamma' : LDB(\mathbf{D}) \rightarrow LDB(\mathbf{V})$, assigning to each legal state $s \in LDB(\mathbf{D})$ a state $\gamma'(s) \in LDB(\mathbf{V})$. (The reason for the "'" notation will become apparent later.)

Regardless of how the system consisting of the base schema and collection of user views is actually implemented, it is necessary to maintain global consistency, in the sense that the state of each view $\Gamma = (\mathbf{V},\gamma)$ should always be the image $\gamma'(s_1)$ of the state s_1 of the base schema. Whenever an update (s_1,s_2) is specified directly on the base schema $\mathbf{D}$, this is no problem, as the new state of view Γ is just $\gamma'(s_2)$. However, when an update (t_1,t_2) is specified by a user directly on Γ, there is a problem of determining an appropriate next state for the base schema. The following definitions formalize the situation.

0.1.2 DEFINITION Let $\mathbf{D}$ be a database schema, and let $\Gamma = (\mathbf{V},\gamma)$ be a view of $\mathbf{D}$.

(a) An *update specification* for Γ is a pair $(s_1,(t_1,t_2))$, where (t_1,t_2) is an update specification to the schema $\mathbf{V}$ and $s_1 \in LDB(\mathbf{D})$ is such that $\gamma'(s_1) = t_1$. (Think of s_1 as the current state of the base schema, and (t_1,t_2) as the update request of the user.)

(b) A *solution* to the update specification $(s_1,(t_1,t_2))$ is an $s_2 \in LDB(\mathbf{D})$ such that $\gamma'(s_2) = t_2$.

(c) An *update strategy* for Γ is a (possibly partial) function $\rho : LDB(\mathbf{D})\times LDB(\mathbf{V}) \rightarrow LDB(\mathbf{D})$. Think of

ρ : (current state of **D**, desired new state of **V**)

$\mapsto$ new state of **D**, if the update is allowed;

undefined if the update is not allowed.

Thus, for each update specification $(s_1,(t_1,t_2))$, ρ gives the solution $\rho(s_1,t_2)$, whenever this value is defined.

0.2 Our Approach in Relation to Existing Results

The problem of determining an appropriate update strategy has been addressed by a number of researchers. Most of the works concern situations in which the base schema is constrained only by simple constraints, such as functional and/or inclusion dependencies ([DaBe78], [DaBe82], [CaAr79], [FuSS79], [Kell82], [CoPa83]). While these works have provided valuable insight into the way in which updates may be reflected, we are interested in establishing general principles of view support, independent of the detailed properties of specific database constraints and mappings.

A much more general approach, which forms the roots of our work, is the constant complement approach of Bancilhon and Spyratos ([Banc79], [BaSp81a], [BaSp81b]). A *join complement* to view Γ_1 is another view Γ_2 such that the combined states of the two views uniquely determine the state of the base schema. In this situation, it is easy to see that for any update specification $(s_1,(t_1,t_2))$ for Γ_1, there can be at most one solution which leaves the state of Γ_2 fixed, so the selection of such a Γ_2 determines uniquely a maximal update strategy on Γ_1. The problem is, of course, how to pick Γ_2. In general, there are many join complements to Γ_1, and the resulting strategy depends upon the choice. Our work picks up where that of Bancilhon and

Spyratos leaves off, in the sense that we develop a "natural" set of views from which the join complements should be chosen. This set of views forms a Boolean algebra under certain natural operations, and is termed the *algebra of components*. We show that by using only members of the algebra of components as join complements, update strategies are unique and independent of the particular complement chosen.

0.3 Outline of the Paper

In section 1, we present a general analysis of the update problem, together with general conditions which any update strategy ought to meet. Also presented is a more detailed development of the constant complement strategy. For this section, we presume a general knowledge of the commonly used language of database theory, such as can be found in [Ullm82] and [Maie83], but no other specialized knowledge.

In section 2, we outline the necessary features of our general algebraic framework for decomposition, which is developed in detail in [Hegn84]. This framework is the cornerstone of our theory, as the Boolean algebra of components is precisely the set of components which arise as the basic units in our decomposition theory.

In section 3, we present the decomposition results, based upon constant complements consisting of components.

1. SURVEY OF PROBLEM CHARACTERISTICS

1.1 The Surjectivity Problem

The issues involved in the surjectivity problem are best illustrated via a simple example.

1.1.1 EXAMPLE Let **D** be the base schema with two binary relation symbols R_{SP} and R_{PJ}, and no constraints whatever. Let **V** be the view with the single ternary relation symbol R_{SPJ}, and let γ be the view mapping which defines the join of R_{SP} and R_{PJ} on the attribute P. A typical instance of this schema and view is shown below.

S	P
s_1	p_1
s_1	p_2
s_2	p_3

R_{SP}

P	J
p_1	j_1
p_1	j_2
p_3	j_1
p_4	j_3

R_{PJ}

S	P	J
s_1	p_1	j_1
s_1	p_1	j_1
s_2	p_3	j_1

R_{SPJ}

Now suppose that the update which inserts the tuple (s_3,p_3,j_3) into the view instance is requested, so that the new state should be instance (a) below. It is easy to see that there is no state of the schema **D** which has this instance as this image, because R_{SPJ} is a join, and hence any image under γ' must satisfy the join dependency *[SP,PJ]. To insert this new tuple, we would have to insert (s_3,p_3) into R_{SP} and (p_3,j_3) into R_{PJ}, which would force the instance of R_{SPJ} to be (b) below.

S	P	J
s_1	p_1	j_1
s_1	p_1	j_2
s_2	p_3	j_1
s_3	p_3	j_3

(a)

S	P	J
s_1	p_1	j_1
s_1	p_1	j_2
s_2	p_3	j_1
s_3	p_3	j_3
s_3	p_3	j_1
s_2	p_3	j_3

(b)

The insertion of (s_3,p_3,j_1) and (s_2,p_3,j_3) into the view is termed a *side effect* in [DaBe78]. In the terminology of [DaBe82], the update has been *performed*, but not *performed exactly*.

The problem here is that we have not endowed the user view with the constraints inherited from the base view. An *implied constraint* of view $\Gamma = (\mathbf{V},\gamma)$ is a constraint on $\mathbf{V}$ which is true for every instance of the form $\gamma'(s)$, with $s \in LDB(\mathbf{D})$. In this case, $\bowtie[SP,PJ]$ is an implied constraint of the view, and should be a consequence of $Con(\mathbf{V})$.

In this work, we shall always assume that the instance mapping $\gamma' : LDB(\mathbf{D}) \dashrightarrow LDB(\mathbf{V})$ of a view $\Gamma = (\mathbf{V},\gamma)$ is surjective. In other words, the constraints of $\mathbf{V}$ are sufficient to forbid any view states which are not images of base schema states. Therefore, it will never be the case that no reflection exists for a given update specification of the view.

Let us remark that it is not always possible to guarantee the surjectivity of a view mapping simply by including first-order implied constraints, even when the base schema is constrained only by functional dependencies, and the view consists entirely of projections. For an example of this situation, consult [Hegn84]. Also, for an elegant and thorough study of the implied constraint problem, see [JaAK82].

1.2 The Admissibility Problem

The admissibility problem is one of selecting an appropriate update strategy. In this section, we present several requirements which any such strategy ought to possess.

REQUIREMENT 1: *An admissible update strategy must not introduce any extraneous components into the new state of the base schema.*

The concept of extraneous update is borrowed from [DaBe78]. Intuitively, an extraneous component of an update reflection is one which is unnecessary to achieve the intended result. An example will make the idea clear.

1.2.1 EXAMPLE Consider again the base and view schemata and example instance of Example 1.1.1. Suppose that we wish to implement the deletion of the tuple (s_1,p_1,j_1) from the view relation R_{SPJ}. This can be accomplished by removing (p_1,j_1) from the instance of R_{PJ}. However, the tuple (p_4,j_3) could also be removed from the instance of R_{PJ}, without altering the effect. The deletion of (p_4,j_3) is an extraneous update, since it is totally unnecessary to achieve the desired effect.

The idea that the reflected changes should be minimal is espoused in [Kell82], and is also central in the general work on the semantics of update in [FaUV83]. The following example illustrates the general idea.

1.2.2 EXAMPLE Consider again the general setting of Example 1.1.1. Suppose

this time that we wish to delete the tuple (s_2,p_3,j_1) from the view relation instance R_{SPJ}. This can be accomplished by deleting either (s_2,p_3) from the instance of R_{SP} or by deleting (p_3,j_1) from the instance of R_{PJ}, or both. There is no minimal change which will realize the desired update, and so we must either make an arbitrary choice, or else delete both tuples. In any case there is no minimal operation on the base state which will accomplish the intended result. However, each of these updates is nonextraneous, as defined below.

1.2.3 NOTATIONAL CONVENTION Given a relational database schema **D**, we define the set operations $\subseteq$, $\cap$, $\cup$, $\setminus$ (difference), and Δ (symmetric difference) in a relation-by-relation fashion. (Recall that $A \Delta B = (A \setminus B) \cup (B \setminus A)$.)

1.2.4 DEFINITION Let $\Gamma = (\mathbf{V},\gamma)$ be a view of schema **D**. (a) Let $(s_1,(t_1,t_2))$ be an update specification for Γ. A solution s_2 to $(s_1,(t_1,t_2))$ is *nonextraneous* (resp. *minimal*) if for any other solution s_3, $s_1 \Delta s_3 \not\subseteq s_1 \Delta s_2$. (resp. $s_1 \Delta s_2 \subseteq s_1 \Delta s_3$).
(b) An update strategy ρ for Γ is *nonextraneous* (resp. *minimal*) if each of its associated defined update strategies is nonextraneous (resp. minimal).

It is tempting to require that an update strategy always have minimal solutions. Unfortunately, this is not possible, as the following example shows.

1.2.5 EXAMPLE Let us, in effect, "turn around" Example 1.1.1. Let **D** be the schema with the single relation symbol R_{SPJ}. Mandate that R_{SPJ} satisfy the join dependency $\bowtie[SP,PJ]$. Let Γ_1 be the view which is defined by the

projection π_{SP} to produce the relation R_{SP}. Similarly, let Γ_2 be the view which is defined by the projection π_{PJ} to produce the relation R_{PJ}. Suppose that the initial states of the schemata are

S	P	J
s_1	p_1	j_1
s_1	p_1	j_2
s_2	p_2	j_2

S	P
s_1	p_1
s_2	p_2

P	J
p_1	j_1
p_1	j_2
p_2	j_2

Now suppose that we wish to insert (s_3,p_1) into the relation of view Γ_1. The most obvious way to do this is to insert (s_3,p_1,j_1) and (s_3,p_1,j_2) into the base relation. However, we could also insert just (s_3,p_1,j_1) into the base relation, and delete (s_1,p_1,j_2). Neither update is extraneous, and so no minimal solution is possible in this case.

We do have the following weaker result on minimal solutions.

1.2.6 PROPOSITION *Let ρ be an update strategy, and suppose that ρ always supplies nonextraneous solutions. Then, whenever a minimal solution exists, ρ will supply that minimal solution.*

Proof: Follows immediately from the fact that if a minimal solution to an update problem exists, then it is the only nonextraneous solution. ∎

REQUIREMENT 2: *An admissible update strategy must be functorial.*

1.2.7 EXAMPLE We again use the schema, view, and initial instances of Example 1.1.1. Suppose that our first update request is to change (s_2,p_3,j_1) to (s_2,p_4,j_1) in the view instance. To do so, we change (s_2,p_3) to (s_2,p_4) in the instance of R_{SP}, and change (p_3,j_1) to (p_4,j_1) in the instance of R_{PJ}. To avoid having (s_2,p_4,j_3) appear in the view, (p_4,j_3) must also be deleted from

the instance of R_{PJ}. It is clear that this defines a minimal update which achieves the desired result. Now suppose that we wish to change (s_2,p_4,j_1) back to (s_2,p_3,j_1). To achieve this, we change (s_2,p_4) back to (s_2,p_3) in the instance of R_{SP}, and change (p_4,j_1) back to (p_3,j_1) in the instance of R_{PJ}. Again, this is a minimal update. Note, however, that the resulting state of the base schema is not the same as the initial state; the result of applying these two changes (which amount to the identity update on the view) does not make the same change on the base view as the identity update (which would make no changes at all to the base instance). Thus, any update strategy with these changes is not functorial in the sense of the following definition.

1.2.8 DEFINITION Let $\Gamma = (\mathbf{V},\gamma)$ be a view of the schema **D**, and let ρ be an update strategy for Γ. ρ is *functorial* if the following two conditions are satisfied.

(a) If $s_1 \in LDB(\mathbf{D})$ and $t_2 \in LDB(\mathbf{V})$ are such that $\gamma'(s_1) = t_2$, then $\rho(s_1,t_2) = s_1$.

(b) If $s_1 \in LDB(\mathbf{D})$ and t_2, $t_3 \in LDB(\mathbf{V})$ are such that $\rho(s_1,t_2)$ and $\rho(\rho(s_1,t_2),t_3)$ are defined, then $\rho(s_1,t_3) = \rho(p(s_1,t_2),t_3)$.

Condition (a) of functoriality just says that the identity update should be reflected in the base schema as no change. Condition (b) says that if we start in base state s_1 and view state t_1, going first to view state t_2 and then to t_3 results in the same base schema state as updating directly from t_1 to t_3. The key property of functoriality is summarized in the following.

1.2.9 OBSERVATION *Let $\Gamma = (\mathbf{V},\gamma)$ be a view of the schema* **D**, *and let ρ be an update strategy for Γ. If ρ is functorial, then for any initial state of the*

base schema and sequence of updates on the view, the final state of the base schema depends only upon the final state of the view schema, and not upon the sequence of updates applied to get there. ∎

REQUIREMENT 3: *An admissible update strategy must be symmetric.*

1.2.10 EXAMPLE Again consider the base schema, view, and initial instances of Example 1.1.1. Suppose we first wish to insert (s_1,p_4,j_4) into the view. To do so minimally, we insert (s_1,p_4) into the instance of R_{SP}, insert (p_4,j_4) into the instance of R_{PJ}, and delete (p_4,j_3) from the instance of R_{PJ}. Now suppose we wish to delete this same tuple from the view. We can delete either (s_1,p_4) or (p_4,j_4) from the appropriate relation to get a nonextraneous deletion. If we seek an update strategy which only allows nonextraneous updates, and which allows the insertion, then the strategy is not symmetric, since the effect of the insertion cannot be undone. This is formalized in the next definition.

1.2.11 DEFINITION Let $\Gamma = (\mathbf{V},\gamma)$ be a view of schema $\mathbf{D}$, and let ρ be an update strategy for Γ. ρ is *symmetric* if whenever $s_1 \in LDB(\mathbf{D})$ and $t_2 \in LDB(\mathbf{V})$ are such that $\rho(s_1,t_2)$ is defined, then $\rho(\rho(s_1,t_2),\gamma'(s_1))$ is also defined.

REQUIREMENT 4 *An update strategy must be state independent.*

EXAMPLE 1.2.12 Consider again the framework and initial states of Example 1.2.5. Suppose we stipulate that all updates to Γ_1 be effected without altering the state of of Γ_2. Suppose that we now want to delete (s_2,p_2) from

the state of view Γ_1. This is impossible without deleting (p_2,j_1) from the state of Γ_2. (We are not allowing null placeholders in this example, so it is not allowable to change (s_2,p_2,j_1) to (null,p_2,j_2); we will address examples later where this is allowed.) Note, however, that if the instance of R_{SPJ} were instead

R_{SPJ}:

S	P	J
s_1	p_1	j_1
s_1	p_1	j_2
s_2	p_2	j_1
s_1	p_2	j_1

,

then the deletion could be performed without changing the state of Γ_2, by deleting (s_2,p_2,j_1), as (p_2,j_1) would remain in the state of the relation of Γ_2 by virtue of the tuple (s_1,p_2,j_1). Hence whether or not an update on a view is allowed depends upon information not visible in the view. This is an undesirable feature, which we rule out.

1.2.13 DEFINITION Let **D** be a database schema, let $\Gamma = (\mathbf{V},\gamma)$ be a view of **D**, and let ρ be an update strategy for Γ. ρ is *state independent* if whenever $\rho(s_1,t_2)$ is defined, then so too is $\rho(r_1,t_2)$ for all r_1 for which $\gamma'(r_1) = \gamma'(s_1)$.

Our reference concept of a good admissible encompasses the requirements which have been stated above.

1.2.14 DEFINITION An update strategy ρ is *admissible* if it is nonextraneous, functorial, symmetric, and state independent.

1.3 The Constant Complement Strategy

In this section, we review the constant complement update strategy of Bancilhon and Spyratos [Banc79], [BaSp81a], and [BaSp81b], with emphasis on ideas pertinent to our framework.

1.3.1 DEFINITION Let **D** be a database schema, and let $\Gamma_1 = (\mathbf{V}_1,\gamma_1)$ and $\Gamma_2 = (\mathbf{V}_2,\gamma_2)$ be views of **D**.

(a) Γ_2 is a *join complement* of Γ_1 (or Γ_1 and Γ_2 are *join complementary*) if the mapping $\gamma_1'\times\gamma_2' : LDB(\mathbf{D}) \rightarrow LDB(\mathbf{V}_1)\times LDB(\mathbf{V}_2)$ defined by $s \mapsto (\gamma_1'(s),\gamma_2'(s))$ is injective. (Bancilhon and Spyratos call this just a *complement*; we append the adjective *join* for reasons which will become apparent later.)

(b) Let $u = (s_1,(t_1,t_2))$ be an update specification for Γ_1, and let s_2 be a solution for u. The solution s_2 to u is performed with Γ_2 *constant* if $\gamma_2'(s_1) = \gamma_2'(s_2)$.

(c) Let ρ be an update strategy for Γ_1. ρ is *constant* on Γ_2 if, for all $s \in LDB(\mathbf{D})$ and $t \in LDB(\mathbf{V}_1)$ such that $\rho(s,t)$ is defined, $\gamma_2'(\rho(s,t)) = \gamma_2'(s)$.

The following theorem, whose proof is immediate from the injectivity of $\gamma_1'\times\gamma_2'$, is the reason for considering updates relative to a complement.

1.3.2 THEOREM *Let* **D** *be a database schema, and let* $\Gamma_i = (\mathbf{V}_i,\gamma_i)$, $i = 1,2$ *be views of* **D**. *Let* $u = (s_1,(t_1,t_2))$ *be an update specification to view* Γ_1. *If* Γ_2 *is a join complement of* Γ_1, *then there is at most one solution to to* u *which leaves* Γ_2 constant. ∎

We can thus solve the problem of deciding which of the many available solutions to an update specification to select by fixing a join complement of the view schema. Furthermore, an update strategy may always be extended to a relatively good one, as the next result shows.

1.3.3 PROPOSITION *Let* **D** *be a database schema and let* $\Gamma_i = (\mathbf{V}_i, \gamma_i)$, $i = 1,2$ *be views of* **D** *which are join complementary. Let* ρ *be an update strategy for* Γ_1 *which is constant on* Γ_2. *Then* ρ *can be extended to an update strategy* $\hat{\rho}$ *which is functorial and symmetric.* ■

The problem now is, of course, how to choose an appropriate join complement. Clearly, the identity view $\mathbf{1} = (\mathbf{D}, 1)$ is a join complement to all views, and no updates at all (other than the identity) can be performed with **1** constant. To avoid such a ludicrous anomaly, Bancilhon and Spyratos suggest using a *minimal* complement; that is, a complement which is "smaller" than any other complement. (We shall make the notion of smaller precise in the next section.) However, they then proceed to show that if one such minimal complement exists, then infinitely many do! Now they allow any function as a database mapping, and we might conjecture that the situation is more pleasant if we restrict attention to mappings defined by more reasonable operations. This is not the case, as we now show.

1.3.4 DEFINITION Let **D** be a database schema, and let $\Gamma_i = (\mathbf{V}_i, \gamma_i)$, $i = 1,2$ be views of **D**.

(a) Γ_1 and Γ_2 are *meet complementary* if the mapping $\gamma_1' \times \gamma_2' : \mathrm{LDB}(\mathbf{D}) \rightarrow \mathrm{LDB}(\mathbf{V}_1) \times \mathrm{LDB}(\mathbf{V}_2)$ is surjective.

(b) Γ_1 and Γ_2 are *complementary* if they are both join complementary and meet

complementary.

1.3.5 OBSERVATION *Let* **D** *be a database schema, and let* $\Gamma_i = (\mathbf{V}_i, \gamma_i)$, $i = 1,2$ *be views of* **D**. *If* Γ_1 *and* Γ_2 *are complementary, then any update to* Γ_1 *is possible with constant complement* Γ_2. *Hence any such update strategy is state independent.*

Proof: By the above definition, $\gamma'_1 \times \gamma_2' : \mathrm{LDB}(\mathbf{D}) \rightarrow \mathrm{LDB}(\mathbf{V}_1) \times \mathrm{LDB}(\mathbf{V}_2)$ is bijective. Hence we may change the value of the first component to any legal value without altering the second. ∎

There are two issues which must be resolved with this approach. First, a complement may not exist. Second, complements may not be unique. Example 1.2.5 provides an example of the former situation, while the next example illustrates the latter.

1.3.6 EXAMPLE Let **D** be the schema with two unary relation symbols R and S, and no constraints whatever. Let $\Gamma_1 = (\mathbf{V}_1, \gamma_1)$ be the schema with the single relation symbol R, with γ_1' the mapping which forgets S and retains the value of the instance of R. Define Γ_2 similarly as the view which retains S and forgets R. Define $\Gamma_3 = (\mathbf{V}_3, \gamma_3)$ as the schema with single unary relation symbol T, with an element in the instance of T iff it is either in the instance of R, or in the instance of S, but not both. An example set of instances is depicted below.

R: A	S: A	R: A	S: A	T: A
a_1	a_1	a_1	a_1	a_2
a_2	a_4	a_2	a_4	a_3
a_3		a_3		a_4
Base Schema		Γ_1	Γ_2	Γ_3

It is straightforward to verify that any two of these views are complementary (both join and meet), and hence none has a uninque minimal complement. Still, Γ_2 seems like a much "nicer" complement of Γ_1 than does Γ_3 for at least two reasons. First of all, it is directly "part" of the base schema, obtained in fact as a sub-schema. Secondly, it allows for more natural update operations. For example, suppose we wish to insert a_4 into the instance of R of the view Γ_1. With constant complement Γ_2, we simply insert a_4 into the instance of R in the schema **D**, and we are done. With constant complement Γ_3, we must keep the instance of T constant, which requires not only inserting a_4 into the instance of R in the main schema, but also deleting a_4 from the instance of S. The former update is minimal, while the latter is not even nonextraneous.

It is the thesis of this work that there are formalizable properties which make the views Γ_1 and Γ_2 in the above example "nicer" than the view Γ_3. In the terminology of the next section, Γ_1 and Γ_2 are *strong views* of **D**, while Γ_3 is not. We shall propose that updates only be performed with respect to join complements which are strong views, and then only under certain further (reasonable) conditions.

2. THE DECOMPOSITION FRAMEWORK

The details of the update theory presented in this paper are based heavily upon the decomposition framework developed in [Hegn83] and [Hegn84]. In this section, the only the most essential features are oulined. For details, including all proofs, the reader is referred in particular to [Hegn84]. We presume a basic familiarity with the language of modern algebra [MaBi67], lattice theory [Grät78], and first-order logic [Ende72]. Also presumed is a basic familiarity with the ways in which first-order logic is used in database modelling (*e.g.*, [Jaco82] or [Hegn84].)

2.1 Schemata, Instances, Morphisms, and Views

A *type algebra* is a triple $\Omega = (\mathbf{T},\mathbf{K},\mathbf{A})$, where:

(a) **T** is a finite set of unary predicate symbols, called the *types*, which form a Boolean algebra under the logical operations of disjunction ($\vee$), conjunction ($\wedge$), and negation ($\neg$). The greatest element of the algebra is the universally true predicate τ_u, while the least element is the universally false predicate τ_ϕ.

(b) **K** is an at most countable set of constant symbols, called the *names*.

(c) **A** is a set of axioms in the first-order language with equality whose (other) relation symbols are precisely those of **T**, and whose constant symbols are precisely those of **K**. The axioms are strong enough to determine, for each $k \in \mathbf{K}$ and each $\tau \in \mathbf{T}$, whether or not $\tau(k)$ is true.

Let $\Omega = (\mathbf{T},\mathbf{K},\mathbf{A})$ be a type algebra. A *type assignment* for Ω is any

model μ of **A**. In other words, a type assignment for Ω is a set X, together with an assignment to each $\tau \in \mathbf{T}$ of a set (= unary relation) $Dom_\mu(\tau)$, and an assignment of an element $k^\mu \in X$ to each $k \in \mathbf{K}$, in a manner consistent with the constraints of **A**.

The type algebra in our framework plays a role which extends that of attributes in the more traditional framework [Maie83]. In fact, we may incorporate attributes in our framework by assuming that for each attribute A, there is a type $\tau_A \in \mathbf{T}$ which defines it. The advantage is that types may interact, wheras attributes do not. For example, if we wish attribute C to be the union of attributes A and B, the axiom $(\forall x)(\tau_C(x) \iff \tau_A(x) \vee \tau_B(x))$ may be used to express this. Such interactions are highly useful in defining horizontal decompositions. Furthermore, we formalize null values as types with exactly one value (usually assigned a constant symbol). For example, $\tau_\eta(\eta) \wedge (\forall x)(\tau_\eta(x) \Longrightarrow x=\eta)$ defines the type τ_η to be the null-value type with null symbol η. We sometimes call η the null value. These are "value inapplicable" nulls; we do not deal with incomplete information "value unknown nulls" in this work.

The type assignment plays a role similar to the attribute domains of the traditional framework. Within a given situation, the type assignment μ is fixed; in particular, *a user is never allowed to update it.* However, to make full use of the results of first-order logic, any type assignment which satisfies the axioms of the type algebra must be allowed.

The rest of our framework is very similar to (the relational part) of database logic of Jacobs [Jaco82]. A *relational schema* (over Ω) is a pair **D** = (Rel(**D**),Con(**D**)), where Rel(**D**) is a finite set of relation symbols, and Con(**D**) is a set of first-order constraints. In general, the elements of Con(**D**) are in the language which includes the symbols of Rel(**D**) and Ω, as well

as equality. A (legal) database instance M for the schema **D** is just a model for both Con(**D**) and the type axioms **A**. If the corresponding submodel of the type axioms is the type assignment μ, then we say that M is *with respect to* μ. The set of all legal instances of **D** with respect to μ is denoted by LDB(**D**,μ), while the class of all legal instances is denoted by LDB(**D**). Because we need to use results from model theory such as Beth's theorem, we do not restrict attention to finite models.

A *database mapping* between schemata $\mathbf{D}_1$ and $\mathbf{D}_2$ is given by an interpretation f of the language of $\mathbf{D}_2$ into the language of $\mathbf{D}_1$, which is the identity on the symbols of the type algebra Ω. Such a morphism is denoted by $f : \mathbf{D}_1 \rightarrow \mathbf{D}_2$. This interpretation defines a function $f' : LDB(\mathbf{D}_1) \rightarrow LDB(\mathbf{D}_2)$ in the usual fashion. Note that by definition, f' preserves the underlying type assignment identically, so that we also have an induced mapping $f'_\mu : LDB(\mathbf{D}_1,\mu) \rightarrow LDB(\mathbf{D}_2,\mu)$ for each type assignment μ.

A *view* of the schema **D** is a pair $\Gamma = (\mathbf{V},\gamma)$, with **V** a schema (over the same type algebra Ω), and $\gamma : \mathbf{D} \rightarrow \mathbf{V}$ a database mapping with the property that the induced mapping $\gamma' : LDB(\mathbf{D}) \rightarrow LDB(\mathbf{V})$ is surjective. View(**D**) denotes the class of all views of the schema **D**.

Before going on, we present an example which illustrates these ideas, and which will be used in later update examples.

2.1.1 EXAMPLE In this example, we show how our framework allows the formalization of null values to permit join dependencies to define exact decompositions. The schema **D** is to have a single 4-ary relation symbol R, with "attributes" ⟨A,B,C,D⟩, and with the join dependency constraint ⋈[AB,BC,CD]. We seek to make the join dependency exact in the sense that nulls will fill in for missing components. Start with four basic types τ_A, τ_B, τ_C, and τ_D,

together with a null type τ_η. The set of types of the type algebra is the free Boolean algebra generated by these five types. η is the sole constant symbol. For convenience, let $\tau_{\bar{A}}$ denote the type $\tau_A \vee \tau_\eta$; define $\tau_{\bar{B}}$, $\tau_{\bar{C}}$, and $\tau_{\bar{D}}$ similarly. The overall "attribute" definition of the relation is then $R[\bar{A},\bar{B},\bar{C},\bar{D}]$, which is expressed as a logical axiom by

$$(\forall x)(\forall y)(\forall z)(\forall w)(R(x,y,z,w) \Longrightarrow \tau_{\bar{A}}(x) \wedge \tau_{\bar{B}}(y) \wedge \tau_{\bar{C}}(z) \wedge \tau_{\bar{D}}(w)).$$

We impose the further constraints that subsumed tuples (those with nulls) are present. More precisely, if a tuple of the form (a,b,c,d) is present, then so too are (a,b,c,η) and (η,b,c,d). If a tuple of the form (a,b,c,η) is present, so too are (a,b,η,η) and (η,b,c,η). If a tuple of the form (η,b,c,d) is present, so too are (η,η,c,d) and (η,b,c,η). It is straightforward to express these rules as first-order constraints (in Con(**D**)); see [Hegn84] for details. The idea of allowing such tuples with nulls in the following manner is not new conceptually; it is the cornerstone of Sciore's notion of *object* [Scio80]. What is different about our presentation is that the rules defining their use are expressible directly in the language of constraints of the schema; there is no need to be informal or extralogical about the use of nulls. This allows us to make deductions of new constraints, as well as determine implied constraints on views, completely within the first-order language we have developed.

Finally, we have the join dependency $\bowtie$[AB,BC,CD] expressed as it is intended for decomposition; namely, if tuples of the form (a,b,η,η), (η,b,c,η), and (η,η,c,d) are present, then so too is a tuple of the form (a,b,c,d). We also have the two embedded join dependencies $\bowtie$[AB,BC] and $\bowtie$[BC,CD]: if tuples of the form (a,b,η,η) and (η,b,c,η) are present, so too is (a,b,c,η); $\bowtie$[BC,CD] is expressed similarly. All are easily converted to first-order sentences.

We have given the maximal representation for the join, in the terminology of [Scio80]. It is straightforward to re-axiomatize to remove subsumed tuples; see [Hegn84] for details. An example instance is given below.

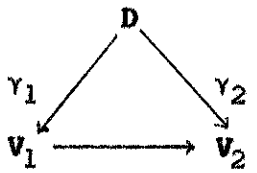
A	B	C	D
a_1	b_1	c_1	d_1
a_1	b_1	c_1	n
a_1	b_1	n	n
n	b_1	c_1	d_1
n	n	c_1	d_1
n	b_1	c_1	n
a_2	b_2	n	n
a_2	b_3	c_3	n
a_2	b_3	n	n
n	b_3	c_3	n
n	n	c_4	d_4

It is particularly convenient to define the component views from this schema. We define the views corresponding to the projections π°_{AB}, π°_{BC}, π°_{CD}, π°_{ABC}, and π°_{BCD} as usual, with the "$\circ$" superscript denoting that we only project the tuples with non-null values in at least two of the projected columns. Denote the corresponding views as $\Gamma^{\circ}_{AB} = (\mathbf{V}^{\circ}_{AB}, \pi^{\circ}_{AB})$, etc. Note that the 3-column views inherit a join dependency constraint, while the two column views inherit only type constraints on their columns. Note in particular that Γ°_{AB} and Γ°_{BCD} (for example) are truly independent (*i.e.*, they are meet complements); this is not so for the AB and BCD projections of an ordinary relation R satisfying $\bowtie$[AB,BCD] because the common column B must match. *We must formalize the null values to get this independence.*

2.2 The Partial Lattice of Views

Let $\Gamma_1 = (\mathbf{V}_1, \gamma_1)$ and $\Gamma_2 = (\mathbf{V}_2, \gamma_2)$ be views of the same schema **D**. A

morphism $f : \Gamma_1 \rightarrow \Gamma_2$ is a database mapping $f : \mathbf{V}_1 \rightarrow \mathbf{V}_2$ such that the following diagram commutes.



f is an *isomorphism* if there is another morphism $g : \Gamma_2 \rightarrow \Gamma_1$ such that $f \circ g$ and $g \circ f$ are each identities. [View(**D**)] denotes the set of equivalence classes of isomorphic views of View(**D**). We note that while View(**D**) is usually a proper class, [View(**D**)] is always a (countable) set.

2.2.1 PROPOSITION *Let* Γ_1 *and* Γ_2 *be views of the same schema* **D**.
(a) *There is at most one morphism* $f : \Gamma_1 \rightarrow \Gamma_2$.
(b) Γ_1 *and* Γ_2 *are isomorphic if and only if there are morphisms* $f : \Gamma_1 \rightarrow \Gamma_2$ *and* $g : \Gamma_2 \rightarrow \Gamma_1$. □

We say that $\Gamma_1 = (\mathbf{V}_1, \gamma_1)$ implicitly defines $\Gamma_2 = (\mathbf{V}_2, \gamma_2)$ if there is a function $h : LDB(\mathbf{V}_1) \rightarrow LDB(\mathbf{V}_2)$ such that $\gamma_2' = f \circ \gamma_1'$. Γ_1 *explicitly defines* Γ_2 if there is a database morphism $f : \Gamma_1 \rightarrow \Gamma_2$. Clearly, if Γ_1 explicitly defines Γ_2, then it implicitly defines it (just take h to be f'). What is remarkable is that the converse holds (thanks to Beth's theorem).

2.2.2 THEOREM *Let* Γ_1 *and* Γ_2 *be views over the same schema* **D**. *Then* Γ_1 *explicitly defines* Γ_2 *if and only if it implicitly defines it.* □.

Let Part(LDB(**D**)) denote the class of all partitions on LDB(**D**) which only identify models based upon the same type assignment. It is easy to see that

Part(LDB(**D**)) forms a (big) lattice with universal bounds. The greatest element is the finest partition on LDB(**D**), while the least element is the partition which identifies, for each type assignment μ, all of the models with respect to μ. Join and meet are just partition supremum and infimum (see [Grät78, Ch. IV, §4]).

With each view $\Gamma = (\mathbf{V},\gamma)$ of **D** we may associate an element $\Pi(\Gamma)$ of Part(LDB(**D**)), namely, the kernel $\ker(\gamma')$ of the mapping $\gamma' : \mathrm{LDB}(\mathbf{D}) \rightarrow \mathrm{LDB}(\mathbf{V})$. (Recall that the *kernel* of $f : A \rightarrow B$ is the equivalence relation on A defined by $x \equiv y$ iff $f(x) = f(y)$.) It is clear that Γ_1 implicitly defines Γ_2 iff $\Pi(\Gamma_1)$ is finer than $\Pi(\Gamma_2)$. Write $\Gamma_2 \preccurlyeq \Gamma_1$ iff Γ_1 defines (explicitly or implicitly) Γ_2. In effect, we have embedded [View(**D**)] into the lattice Part(LDB(**D**)). Hence $\preccurlyeq$ is a partial ordering on [View(**D**)]. Note that this embedding is nowhere near surjective. While Part(LDB(**D**)) is a proper class and not a set, in general, [View(**D**)] is a countable set. Hence there are lots of "holes" in the embedding. In other words, very few of the partitions of LDB(**D**) arise as the result of database mappings. It is primarily on this issue that we differ with the modelling philosophy of Bancilhon and Spyratos [Banc79], [BaSp81a], [BaSp81b]. While they consider *any* partition to legitimately define a view, we only consider those (by comparison, very few) which arise as logical interpretations.

Joins and meets of views are defined as in Part(LDB(**D**)). It is *not* the case that joins and meets of views always exist as views; in other words, [View(**D**)] does not form a sublattice of Part(LDB(**D**)), but only a *relative (partial) sublattice* [Grät78, p. 40]. Note, however, that the finest partition corresponds to the identity view $\mathbf{1}_{\mathbf{D}}$, and the coarsest partition corresponds to the zero view $\mathbf{0}_{\mathbf{D}}$, and so we may speak of complements within [View(**D**)]. (The *zero view* preserves the type assignment μ, but contains no

other relations at all.) Referring back to our terminology of Definitions 1.3.1 and 1.3.4, it is easy to see that Γ_2 is a join complement of Γ_1 if $\Gamma_1 \vee \Gamma_2 = \mathbf{1_D}$, and that it is a meet complement if $\Gamma_1 \wedge \Gamma_2 = \mathbf{0_D}$. A complementary view then just reduces to a complement in the partial lattice [View(**D**)]. Example 1.3.6 shows that complements in this partial lattice need not be unique.

2.3 Strong Views and their Relatives

Example 1.3.6 shows that some views seem to be better behaved than others. The formalization of strong views and strongly complemented strong views is an attempt to formalize this behavior. The concepts have a strong order-theoretic flavor, and we first state them independently of database concepts.

A $\perp$-*poset* is a partially ordered set $\mathbf{P} = (P, \leq)$ with a least element $\perp$. A mapping $f : P \rightarrow Q$ between $\perp$-posets $\mathbf{P} = (P, \leq)$ and $\mathbf{Q} = (Q, \leq)$ is a *morphism* if it is monotonic and $f(\perp) = \perp$. $\langle \mathbf{P} \rightarrow \mathbf{Q} \rangle$ denotes the set of all morphisms from **P** to **Q**. $\langle \mathbf{P} \rightarrow \mathbf{Q} \rangle$ clearly forms a $\perp$-poset under the ordering $f \leq g$ iff $f(x) \leq g(x)$ for all $x \in P$. The least element is the function $\perp$ which sends each x to $\perp$.

Under the relation-by-relation inclusion of 1.2.3, LDB(**D**,μ) forms a poset for each type assignment μ. We say that **D** has the *null model property* if the empty model (*i.e., the model in which the instance of each* $R \in$ Rel(**D**) is empty is in LDB(**D**,μ), for each type assignment μ. If **D** has the null model property, then this null model is the least element in the ordering of LDB(**D**,μ); hence it is a $\perp$-poset. (Actually, it suffices for each LDB(**D**,μ) to

contain a minimal model which is definable by a single first-order sentence.)

The idea here is to exploit the *algebraic* properties of a view mapping γ to ensure that it has nice properties. We first characterize these properties in terms of $\perp$-posets.

Let $\mathbf{P} = (P,\leq)$ and $\mathbf{Q} = (Q,\leq)$ be $\perp$-posets, and let $f \in \langle \mathbf{P} \to \mathbf{Q} \rangle$. f *admits least preimages* if for any $y \in f(P)$, there is a least $x \in P$ such that $f(x) = y$. Denote the least preimage of y by y_f. $lp(f)$ denotes the set of all $x \in P$ with least preimages. f is *least right invertible* if it is surjective, admits least preimages, and the function $f^{\#} : Q \to P$ $y \mapsto y_f$ is in $\langle Q \to P \rangle$. The morphism $f^{\#} \circ f \in \langle \mathbf{P} \to \mathbf{P} \rangle$ is called the *endomorphism* of f, and is denoted $f^{@}$. f is *downward stationary* if whenever $x \in lp(f)$ and $y \leq x$, then $y \in lp(f)$ also. f is a *strong morphism* (of $\perp$-posets) if it is downward stationary and least right invertible. $\langle \mathbf{P} \twoheadrightarrow \mathbf{Q} \rangle$ denotes the set of all strong morphisms from $\mathbf{P}$ to $\mathbf{Q}$.

Let $\mathbf{P} = (P,\leq)$ again be a $\perp$-poset. A morphism $f \in \langle \mathbf{P} \to \mathbf{P} \rangle$ is called a *strong endomorphism* if it is idempotent (*i.e.* $f = f \circ f$) and downward stationary. The set of all strong endomorphisms on $\mathbf{P}$ is denoted $\langle\langle \mathbf{P} \to \mathbf{P} \rangle\rangle$.

The relationship between strong morphisms and strong endomorphisms is a close one, as the following result shows.

2.3.1 LEMMA *Let* $\mathbf{P} = (P,\leq)$ *and* $\mathbf{Q} = (Q,\leq)$ *be* $\perp$*-posets.*
(a) *If* $f \in \langle \mathbf{P} \twoheadrightarrow \mathbf{Q} \rangle$, *then* $f^{@} \in \langle\langle \mathbf{P} \to \mathbf{P}. \rangle\rangle$
(b) *If* $g \in \langle\langle \mathbf{P} \to \mathbf{P} \rangle\rangle$, *then the mapping* $f^{\$} : P \to f(P)$ *is in* $\langle \mathbf{P} \twoheadrightarrow \mathbf{Q} \rangle$.
(Here $f(P)$ *is the underlying set of the sub-*$\perp$*-poset* $f(\mathbf{P}) = (f(P),\leq)$ *of* $\mathbf{P}$.*)* □

It is easy to see that $\langle\langle \mathbf{P} \to \mathbf{P} \rangle\rangle$ forms a partially ordered set with universal bounds under the ordering and least element inherited from $\langle \mathbf{P} \to \mathbf{P} \rangle$;

the greatest element is the identity function 1_P. What is more significant is the following.

2.3.2 LEMMA *Let* **P** *be a* $\perp$*-poset.*

(a) *Complements are unique in* $\langle\langle \mathbf{P} \to \mathbf{P} \rangle\rangle$, *and the complemented elements form a Boolean algebra.*

(b) *If* f *and* g *are complements in* $\langle\langle \mathbf{P} \to \mathbf{P} \rangle\rangle$, *then the mapping* $f \times g : P \to f(P) \times g(P)$ *is a* $\perp$*-poset isomorphism. (Here* $f(P) \times g(P) = (f(P) \times g(P), \leq)$ *is a* $\perp$*-poset under the obvious operations.)* □

These notions translate directly to database mappings and views. The database mapping $f : \mathbf{D}_1 \to \mathbf{D}_2$ is *strong* if $f'_\mu : LDB(\mathbf{D}_1, \mu) \to LDB(\mathbf{D}_2, \mu)$ is a strong morphism (of $\perp$-posets) for each type assignment μ. The view $\Gamma = (\mathbf{V}, \gamma)$ is *strong* if γ is a strong database mapping. Similarly, a database mapping $f : \mathbf{D}_1 \to \mathbf{D}_1$ is a *strong endomorphism* if f'_μ is a strong endomorphism (of $\perp$-posets) for each type assignment μ. A *strong view* of the schema **D** is a view $\Gamma = (\mathbf{V}, \gamma)$ for which γ is a strong morphism. Γ is *strongly complemented* if it is a strong view with a complement which is also a strong view.

The superscripts @, #, and $ are used on database mappings with their same meaning as in $\perp$-poset morphisms. For example, if $f : \mathbf{D}_1 \to \mathbf{D}_2$ is a strong database mapping, then $f^{\#} : \mathbf{D}_2 \to \mathbf{D}_1$ denotes its least right inverse, defined as the unique database mapping given for each type assignment μ by the least right inverse of f'_μ.

2.3.3 THEOREM *Let* **D** *be a database schema with the null model property, and let* $\Gamma_1 = (\mathbf{V}_1, \gamma_1)$ *and* $\Gamma_2 = (\mathbf{V}_2, \gamma_2)$ *be strong views of* **D**.

(a) $\Gamma_1 \preccurlyeq \Gamma_2$ *(as elements of* [View(**D**)]*) iff for each type assignment* μ, $(\gamma_1)^!_\mu \leq (\gamma_2)^!_\mu$ *(as elements of the* $\perp$*-poset* $\langle \mathrm{LDB}(\mathbf{D},\mu) \rightarrow \mathrm{LDB}(\mathbf{D},\mu) \rangle$*).*

(b) *If* $(\gamma_1)^{!@}_\mu$ *is a complement of* $(\gamma_2)^{!@}_\mu$ *in* $\langle\langle \mathrm{LDB}(\mathbf{D}) \rightarrow \mathrm{LDB}(\mathbf{D}) \rangle\rangle$ *for each type assignment* μ*, then* Γ_1 *is a complement of* Γ_2 *in* [View(**D**)]*. Furthermore,* Γ_2 *can have at most one complement of this form. Thus, strong complements are unique.*

□

It follows from 2.3.2(a) and 2.3.3(b) that the strongly complemented strong views form a Boolean algebra, as a subalgebra of [View(**D**)]. We call this algebra the *component algebra* of **D**. The strong complement of $\Gamma = (\mathbf{V},\gamma)$ is denoted $\Gamma^C = (\mathbf{V}^C,\gamma^C)$. It also follows from 2.3.1 that a strong view $\Gamma = (\mathbf{V},\gamma)$ is entirely determined by its endomorphism $\gamma^@$. In fact, if the schema **D** is endowed with all of the implied constraints of the mapping $\gamma^@ : \mathbf{D} \rightarrow \mathbf{D}$, yielding $\mathbf{D}^\$$, we get a view $\Gamma^\$ = (\mathbf{D}^\$,\gamma^\$)$ which is isomorphic to Γ, but which has the same relation symbols as the base schema. An example will illustrate.

2.3.4 EXAMPLE Return to the setting of Example 2.1.1. Each of the Γ° views defined there is strong. Let us consider specifically Γ°_{AB}. Monotonicity and $\perp$ preservation are obvious. For any relation instance of the view, the least preimage is the base instance which appends nulls to the last two columns. An example is given below.

A	B
a_1	b_1
a_2	b_2

View instance

A	B	C	D
a_1	b_1	n	n
a_2	b_2	n	n

Base Schema Instance

The least right inverse $(\pi^\circ_{AB})^\#$ is defined by this association. Downward

stationarity is obvious, as any base relation for which all tuples have nulls in the last two columns is a least preimage.

The database mapping $(\pi^{\circ}_{AB})^{@} : \mathrm{LDB}(\mathbf{D}) \longrightarrow \mathrm{LDB}(\mathbf{D})$ is just the restriction of the base relation to tuples with nulls in the last two columns. Formally, if R_b denotes the relation symbol of the base schema, and R_v the relation symbol of the view, then the defining formula is

$$(\forall x)(\forall y)(\forall z)(\forall w)(R_v(x,y,z,w) \Longleftrightarrow R_b(x,y,z,w) \wedge \tau_A(x) \wedge \tau_B(y) \wedge \tau_\eta(z) \wedge \tau_\eta(w))$$

We call such a view a *restriction* or *object*; the defining database mapping is denoted $\rho(R(\tau_A, \tau_B, \tau_\eta, \tau_\eta))$. Such a mapping is considered to be a generalization of Sciore's notion of object [Scio80]. It is precisely this idea that the view (= "quotient mapping") may be regarded as a subschema (= "immersion") that characterizes strong views. (The mathematical generalization here is that of a component of a direct sum; if X is some algebraic object and $X = X_1 \oplus X_2$, then we have both a projection $X \longrightarrow X_1$ and an injection $X_1 \longrightarrow X$. The same idea is working here; we think of the base relation R[ABCD] as being the "direct sum" (= join) of the components R[AB], R[BC], and R[CD]. For a much more detailed discussion of the aspects of algebraic decomposition, see [Hegn84].)

To get $(\Gamma^{\circ}_{AB})^{\$}$, we enforce (on $\mathbf{D}^{\$}$) the constraint that the C and D columns contain only nulls.

The component algebra is generated by Γ°_{AB}, Γ°_{BC}, Γ°_{CD}. The other members are then $\mathbf{1}_{\mathbf{D}}$, $\mathbf{0}_{\mathbf{D}}$, Γ°_{ABC}, Γ°_{BCD}, and $\Gamma^{\circ}_{AB} \vee \Gamma^{\circ}_{CD}$. The last view consists of two relations, one the AB projection and the other the CD projection. Its endomorphism projects down all tuples of the form (a,b,η,η) and (η,η,c,d).

The strong complement of Γ°_{AB} is Γ°_{BCD}; this is easily verified. Other complements are computed similarly. However, we should remark that it is possible to produce examples of strong views which do not have strong comple-

ments.

A view which is isomorphic to a strong view (resp. strongly complemented strong view) is called a *generalized strong view* (resp. *generalized strongly complemented strong view*). Most of our subsequent results carry over to this more general case.

3. VIEW UPDATING USING STRONG JOIN COMPLEMENTS

All of the results of this section depend upon the condition that the base schema have the null model property. Therefore, *for all of this section,* **D** *is assumed to be a relational database schema with the null model property. We also assume that an update leaves the type assignment fixed.*

3.1 Updating Strongly Complemented Strong Views.

Updating a strongly complemented strong view is the cleanest situation, as updates are always possible, and are always admissible. As this situation forms the basis for the analysis of more complex cases, we consider it first. We know from section 1 that updates relative to complements are quite well behaved; what we get in this case is the added property of nonextraneousness.

3.1.1 THEOREM *Let* $\Gamma_1 = (\mathbf{V}_1, \gamma_1)$ *be a strong view of* **D**, *and let* $\Gamma_2 = (\mathbf{V}_2, \gamma_2)$ *be a strong complement of* **D**. *Then any update specification* $S = (s_1, (t_1, t_2))$ *to* Γ_1 *has a unique solution with constant complement* Γ_1. *This solution is admissible.*

Proof outline: By Theorem 1.3.2, the solution is unique, and by Proposition 1.3.3, functorial and symmetric. By Theorem 1.3.5, any update is possible. Hence, to show admissibility, all we need establish is that the solution is nonextraneous. By 2.3.4, there is a $\perp$-poset isomorphism $h = (\gamma_1^{@})_\mu' \times (\gamma_2^{@})_\mu' : LDB(\mathbf{D},\mu) \rightarrow (\gamma_1^{@})_\mu'(LDB(\mathbf{D},\mu)) \times (\gamma_2^{@})_\mu'(LDB(\mathbf{D},\mu)))$. Clearly, an update to Γ_1 with constant complement Γ_2 will only change the second component; that is, if $h(s_1) = (t_1, s_2)$ and r is the unique solution with constant complement, then

$h(r) = (t_2, s_2)$. Now suppose that p is a solution to S with $p \Delta s \subseteq r \Delta s$. Let $(t_2, p_2) = h(p)$. Since $(\gamma_2^{@})_U'$ is idempotent and downward stationary, $p_2 \subseteq p$, $s_2 \subseteq r$, and $s_2 \subseteq s$. Hence $p_2 \subseteq s_2$ (else $p \Delta s \not\subseteq r \Delta s$). Similarly, $s_2 \subseteq p_2$, so that $s_2 = p_2$. Hence r is a nonextraneous solution. ∎

3.2 Updating Arbitrary Views

Let Γ_1 be any view of **D** (not necessarily strong). A strongly complemented strong view Γ_2 is a *strong join complement* of Γ_1 if $\Gamma_2^c \preccurlyeq \Gamma_1$.

3.2.1 LEMMA *Let* **D** *be a database schema with the null model property, and let* Γ_1 *and* Γ_2 *be views of* **D**, *with* Γ_2 *strong.*

(a) *If* Γ_2 *is a strong join complement of* Γ_1, *then it is also a join complement (in the sense that* $\Gamma_1 \vee \Gamma_2 = \mathbf{1}_D$ *in* [View(**D**)]*).*

Proof: (a) $\Gamma_2^c \preccurlyeq \Gamma_1$ means that $\Pi(\Gamma_2^c)$ is coarser than $\Pi(\Gamma_1)$, and since $\sup(\Pi(\Gamma_2^c), \Pi(\Gamma_2))$ is the identity partition, surely $\sup(\Pi(\Gamma_1), \Pi(\Gamma_2))$ is also the identity partition. ∎

3.2.2 MAIN UPDATE THEOREM *Let* Γ_1 *be any view of* **D** *and let* $S = (s_1, (t_1, u_1))$ *be any update specfication on* Γ_1.

(a) *Let* Γ_2 *be a strong join complement of* Γ_1. *If* S *has a solution with constant complement* Γ_2, *then this solution is admissible.*

(b) *If* Γ_2 *and* Γ_3 *are each strong join complements of* Γ_1 *for which* S *has a solution, then the solution is the same in each case. In other words, if an update specification is performed with respect to a join complement which is a strong view and whose complement is smaller than* Γ_1, *then the solution of the*

update is independent of the particular complement chosen.

Proof outline: (a) Let s_2 be the unique solution to S. Since Γ_2 is a strong join complement of Γ_1, $\Gamma_2{}^c \preccurlyeq \Gamma_1$, and so there is a surjective database morphism $f : \Gamma_1 \rightarrow \Gamma_2{}^c$, by 2.2.2. Hence s_2 must also be a solution to the update specification $(s_1,(f'(t_1),f'(t_2))$ which is guaranteed to be admissible by Theorem 3.1.1.

(b) Suppose that S has a solution for both constant complement Γ_2 and constant complement Γ_3. Denote the solutions to S for these complements by s_2 and s_3, respectively. First, note that $(\gamma_2 \wedge \gamma_3)'(s_2) = (\gamma_2 \wedge \gamma_3)'(s_3) = (\gamma_2 \wedge \gamma_3)'(s_1)$, just by definition of constant complement. Next, $(\gamma_2{}^c)'(s_2) = (\gamma_2{}^c)'(s_3)$, since $\gamma_1'(s_2) = \gamma_1'(s_3)$, and $\Gamma_2{}^c \preccurlyeq \Gamma_1$. Similarly, $(\gamma_3{}^c)'(s_2) = (\gamma_3{}^c)'(s_3)$. Hence $(\gamma_2{}^c \vee \gamma_3{}^c)'(s_2) = (\gamma_2{}^c \vee \gamma_3{}^c)(s_3)$. Thus, $s_2 = s_3$. ■

3.2.3 UPDATE PROCEDURE To service an update specification $(s_1,(t_1,t_2))$ to schema $\Gamma_1 = (\mathbf{V}_1,\gamma_1)$ with strong complement $\Gamma_2 = (\mathbf{V}_2,\gamma_2)$, proceed as follows. Let $f : \Gamma_1 \rightarrow \Gamma_2{}^c$ be the unique view morphism, guaranteed to exist because $\Gamma_2{}^c \preccurlyeq \Gamma_1$. The first step is to determine the solution s_2 to the update specification $(s_1,(f'(t_1),f'(t_2)))$ to $\Gamma_2{}^c$ which exists uniquely and nicely, by 3.1.1. Next, determine $\gamma_1'(s_2)$. If this is the update requested (*i.e.*, t_2), we have succeeded. Otherwise, the update is not possible with constant complement Γ_2.

Note that the update procedure has much in common with the query modification strategy of Stonebraker [Ston75]. We illustrate further with an example.

3.2.4 EXAMPLE We use the framework of Example 2.1.1. Consider the view Γ_{ABD}

$= (\mathbf{V}_{ABD}, \pi_{ABD})$, which just projects the three columns of the base relation, with no regard for types or nulls. Thus, the view state corresponding to the example state of 2.1.1 will be

A	B	D
a_1	b_1	d_1
a_1	b_1	η
η	b_1	d_1
η	η	d_1
η	b_1	η
a_2	b_2	η
a_2	b_3	η
η	b_3	η
η	η	d_4

The constraints may be summarized by stating that (i) for any tuple of the form (a,b,d), there are also tuples of the form (a,b,η) and (η,b,d); (ii) for any tuple of the form (η,b,d), there are also tuples of the form (η,b,η) and (η,η,d); (iii) there are no tuples of the form (a,η,d),(a,η,η), or (η,η,η).

The smallest strong join complement of Γ is Γ°_{BCD}. Therefore all updates must be "filtered" through the complementary view Γ°_{AB}. In other words, the only updates which can be performed are those which could be achieved by updating the state of Γ°_{AB}. Let $f : \Gamma \rightarrow \Gamma^{\circ}_{AB}$ be the unique view morphism, guaranteed because $\Gamma^{\circ}_{AB} \preccurlyeq \Gamma$. f' is easily seen to be the mapping which first projects the A and B columns (dropping D), and then saves only those tuples without η entries.

Suppose that the update request is to delete (a_2,b_3,η) and (η,b_3,η). This maps to deleting (a,b) from $\mathbf{V}_{AB}$, which achieves the desired effect, so that the update is allowed. On the other hand, suppose that the update request is to delete (η,η,d_4). (Note that the resulting state of the view is legal.) This maps to doing nothing in $\mathbf{V}_{AB}$; hence the update cannot be effected with constant complement Γ_2.

3.3 Updating Strong Views

The need to update with respect to a strong join complement, and not just a join complement, is crucial, as the next example shows.

3.3.1 EXAMPLE Refer back to Example 1.3.6. It is clear that Γ_1 and Γ_2 are strongly complemented strong views and each other's complement in the component algebra. Γ_3 is also a complement of each, although it is not even a strong view. Suppose we update Γ_1 with constant complement Γ_3. Specifically, suppose insert a_4 into the state of Γ_1. As shown in Example 1.3.6, this produces an extraneous deletion of a_4 in S, violating the admissibility of the update.

The problem illustrated in the above example cannot occur if the view to be updated is a strong view. We defer the proof of the following result to a full paper, as it uses concepts from [Hegn84] which we have not discussed here.

3.3.1 LEMMA *Let Γ_1 and Γ_2 be strong views of* **D**, *and assume further that Γ_1 is strongly complemented. Then for Γ_1 to be a strong join complement of Γ_2, it suffices that it be an ordinary join complement.* □

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