A Simple Counterexample to the Finite Axiomatizability of Relational Views

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It is part of the folklore of the theory of relational databases that there exists a schema with a simple axiomatization which has a projective view which is not finitely axiomatizable. From time to time, it is useful to be able to point to such an example, complete with an explanation of why finite axiomatizability fails. Unfortunately, they have rarely made it into the literature. Only two are known to the author. In [Hul84, Lemma 4.1], Hull presents an example of a schema with five attributes constrained by three functional dependencies (FDs). The view which is not finitely axiomatizable is a projection onto four of those attributes. In [Heg90], the author identifies a simpler example containing just four attributes and constrained by three FDs, with the corresponding view a projection onto three of those attributes. Unfortunately, that example contains an error; and, in any case, no argument for its validity is offered. In this note, the example of [Heg90] is corrected, and the proof of the lack of finite axiomatizability of the view is elaborated.

Let $\mathbf{E}_1$ be the relational schema with the single relation name $R[ABCD]$ on four attributes; the constraining set of FDs is $\mathcal{F}_1 = \{A \to D, B \to D, CD \to A\}$. The domain of possible values for each attribute is assumed to be infinite. Let $\Pi_{ABC} = (R[ABC], \pi_{ABC})$ denote the view which is the projection onto the attributes ABC. For any $n > 0$, let $r(n)$ denote the instance which is depicted in Fig. 1. Assume that any two elements with distinct names are distinct values, save that a_1 and a_n may be the same. It is easy to see that $r(n)$ is a legal instance of the main schema if and only if $a_1 = a_n$. Similarly, $r'(n) = \pi_{ABC}(r(n))$ is a legal instance of Π_{ABC} under the implied constraints if and only if $a_1 = a_n$, since a simple "chase" through any element of $\pi^{-1}_{ABC}(r'(n))$ shows that all of the values in the column of attribute D must be the same. However, if any tuple from $r'(n)$ is deleted, a valid instance of $R[ABC]$ is obtained even if $a_1 \neq a_n$, since it is now possible to have two distinct values appearing in column D in an inverse image. This situation is shown in Fig. 2 with the row containing (a_3, b_4, c_6) deleted. In this case, it need not be the case that $a_1 = a_0$. Therefore, Π_{ABC} is not axiomatizable by any set of sentences having only n free tuple variables (*i.e.*; a maximum of n variables per column). Since n is arbitrary, Π_{ABC} is not finitely axiomatizable. In particular, it is not axiomatizable by any finite set of equality generating dependencies (EGDs) [AHV95, 10.1], much less by a family of functional dependencies.

$$\begin{bmatrix}
a_1 & b_1 & c_1 & d_1 \\
a_1 & b_2 & c_2 & d_1 \\
a_2 & b_2 & c_3 & d_1 \\
a_2 & b_3 & c_4 & d_1 \\
a_3 & b_3 & c_5 & d_1 \\
a_3 & b_4 & c_6 & d_1 \\
a_4 & b_4 & c_7 & d_1 \\
a_4 & b_5 & c_8 & d_1 \\
\vdots & \vdots & \vdots & \vdots \\
a_{k-1} & b_{k-1} & c_{2k-3} & d_1 \\
a_{k-1} & b_k & c_{2k-2} & d_1 \\
\vdots & \vdots & \vdots & \vdots \\
a_{n-1} & b_{n-1} & c_{2n-3} & d_1 \\
a_{n-1} & b_n & c_{2n-2} & d_1 \\
a_n & b_n & c_1 & d_1
\end{bmatrix}$$

Figure 1: The layout of the generic counterexample instance

$$\begin{bmatrix}
a_1 & b_1 & c_1 & d_1 \\
a_1 & b_2 & c_2 & d_1 \\
a_2 & b_2 & c_3 & d_1 \\
a_2 & b_3 & c_4 & d_1 \\
a_3 & b_3 & c_5 & d_1 \\
\cancel{a_3} & \cancel{b_4} & \cancel{c_6} & \cancel{\quad} \\
a_4 & b_4 & c_7 & d_2 \\
a_4 & b_5 & c_8 & d_2 \\
\vdots & \vdots & \vdots & \vdots \\
a_{k-1} & b_{k-1} & c_{2k-3} & d_2 \\
a_{k-1} & b_k & c_{2k-2} & d_2 \\
\vdots & \vdots & \vdots & \vdots \\
a_{n-1} & b_{n-1} & c_{2n-3} & d_2 \\
a_{n-1} & b_n & c_{2n-2} & d_2 \\
a_n & b_n & c_1 & d_2
\end{bmatrix}$$

Figure 2: The layout of the generic counterexample instance with one row deleted

References

[AHV95] Abiteboul, S., Hull, R., and Vianu, V., *Foundations of Databases*, Addison-Wesley, 1995.

[Heg90] Hegner, S. J., "Some open problems on view axiomatization," *Bulletin of the EATCS*, **40**(1990), pp. 496–498.

[Hul84] Hull, R., “Finitely specifiable implicational dependency families,” *J. Assoc. Comp. Mach.*, **31**(1984), pp. 210–226.