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S. J. Hegner

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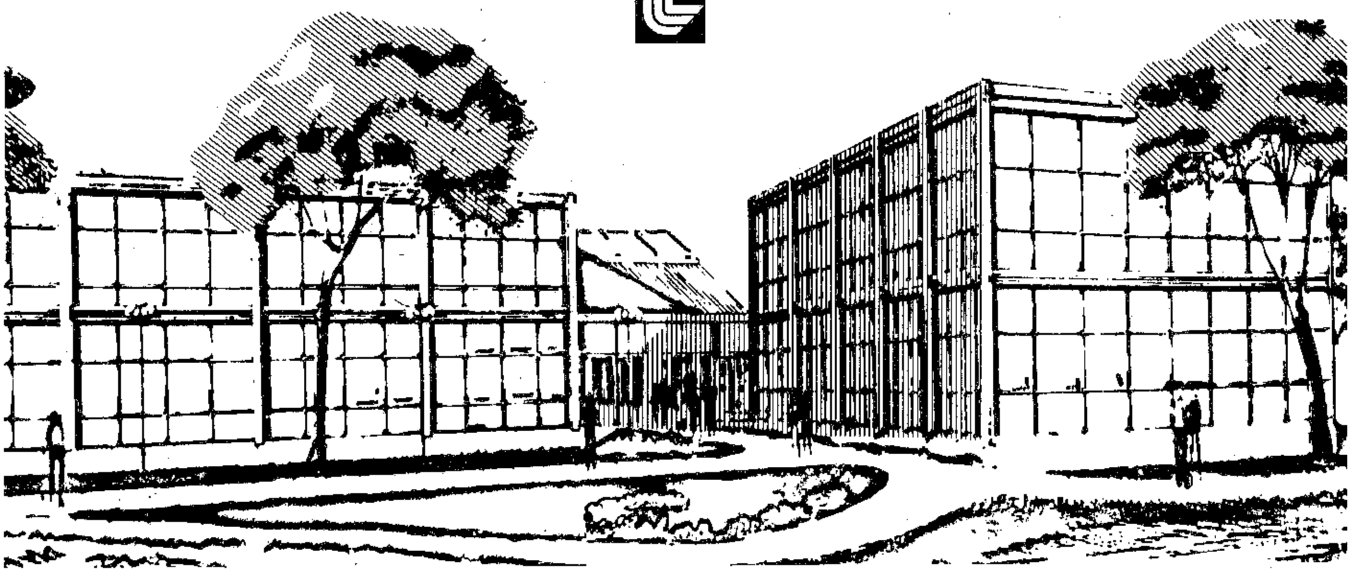


STRUCTURED-ACTION APPROACH TO CONTINUOUS-TIME SYSTEMS

Stephen J. Hegner
University of California, Lawrence Livermore Laboratory
P.O. Box 808, L-156, Livermore, California 94550

Abstract

A smooth system in a category of locally-convex spaces K is given by f: $Q \to Q$, g: $I \to Q$, h: $Q \to Y$, with f the infinitesimal generator of a differentiable semigroup. Using the theory of categories relative to the category of locally-convex spaces, a general theory of behavior, realization, and duality for smooth systems in K is given, for certain chosen categories K.

1. Introduction

This article is an extended abstract without proofs. Full details will appear in a forthcoming report [8].

In recent years there has been a substantial amount of work directed towards using category theory as a tool for describing the basic ideas in system theory, most of it having been in discrete-time systems. See, for example [1], [3], and [11]. However, the author recently showed that categorical techniques are applicable as well to continuous-time systems [7]. In that report, only ordinary category theory was used, and consequently some of the constructions were relatively cumbersome. In this report, it is shown that by using relative (to the category of locally-convex spaces) category theory, the basic results in continuous-time systems may be obtained as easily and elegantly as their discrete-time counterparts. The classical results in behavior, realization, and duality are generalized substantially, while important questions in the canonical structure of behavior and in duality of infinite-dimensional systems are resolved.

2. Categories Relative to LCS

Let $\underline{K}$ be the field of real numbers $\underline{R}$ or the field of complex numbers $\underline{C}$. Denote by LCS the category whose objects are the locally-convex topological vector spaces (l.c.s.'s) over $\underline{K}$ and whose morphisms are the continuous linear maps. An LCS category is an ordinary category K subject to the additional conditions that (a) Each morphism set $K(E,F)$ has the structure of a l.c.s. (b) Morphism composition $K(E,F) \times K(F,G) \to K(E,G)$ is bilinear and separately continuous. (An LCS category is a special case of the general construction of a category relative to a closed monoidal category [2, Ch. 9]. Full details relating relative category theory to this approach will appear in [8].)

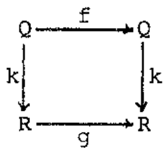
Let K be any full subcategory of LCS. K may be regarded as an LCS category in a natural way. For any pair (E,F) of l.c.s.'s, let $L_s(E,F)$ denote the space of all continuous linear maps $E \to F$ with the topology of pointwise convergence. It is easy to verify the conditions (a) and (b) above for this structure, which will always be assumed for subcategories of LCS used in this article. The opposite category K^{op} of K is also an LCS category in a natural way. Namely, since as sets $K(E,F) = K^{op}(F,E)$, the l.c.s. structure of $K^{op}(F,E)$ is just that of $K(E,F)$.

Let K and L be LCS categories, and let P: $K \to L$ be a functor. P is an LCS functor if each morphism function P_{EF}: $K(E,F) \to L(P(E),P(F))$ is a continuous linear map. P is an LCS equivalence if each P_{EF} is an isomorphism of l.c.s.'s.

A natural transformation of LCS functors is an LCS natural transformation without additional requirements. Let K be a one-object LCS category and let L be any LCS category. The LCS functor category $[K,L]$ has as objects all LCS functors $K \to L$ and natural transformations as morphisms. For P,Q: $K \to L$ LCS functors, the LCS structure on $[K,L](P,Q)$ is that induced as a subspace of $L_s(P(1),Q(1))$, where 1 is the unique object of K.

3. Differentiable Semigroups and Systems

In the discrete-time case of linear systems, a system dynamics in a category K is a pair (Q,f), where Q is a K object and $f \in K(Q,Q)$ [1]. A morphism of dynamics is called a dynamorphism. $k \in K(Q,R)$ is a dynamorphism $(Q,f) \to (R,g)$ if the diagram



commutes. This definition of morphism makes system dynamics in K into a category $\mathrm{Dyn}(K)$. A decomposable system in K is a 6-tuple $M = (Q,f,I,g,Y,h)$ with (Q,f) a system dynamics (Q is called the state-space and f the state-transition map), I is a K object (the input space), $g \in K(I,Q)$ (the input map), Y is a K object (the output space), and $h \in K(Q,Y)$ (the output map). When is a category of vector spaces and linear maps, the system is thought of as described by

$$q(t+1) = f(q(t)) + g(i(t))$$
$$y(t) = h(q(t)).$$

In continuous time, the one-step transition is replaced by an infinitesimal transition. That is, the above equations now become

$$\frac{dq(t)}{dt} = f(q(t)) + g(i(t)) \qquad (*)$$
$$y(t) = h(q(t))$$

To insure that the equations (*) are meaningful, the differential equation must be solvable. This requires that the dynamics (Q,f) be of a special nature, namely that f be the infinitesimal generator of a differentiable semigroup.

Let E be a l.c.s., let $\underline{\underline{R}}_+$ be the nonnegative reals, and let $L(E)$ denote the space of continuous endomorphisms of E. A (weak) differentiable semigroup (d.s.g.) on E is a map $T: \underline{\underline{R}}_+ \to L(E)$ such that (a) $T(0) = 1_E$, (b) $T(s+t) = T(s) \circ T(t)$ for all $s,t \in \underline{\underline{R}}_+$, (c) $\lim_{t \to 0} \frac{T(t)e-e}{t}$ exists for all $e \in E$. Define $g_T: E \to E$ by $g_T(e) = \lim_{t \to 0} \frac{T(t)e-e}{t}$. It is easy to see that $g_T \in L(E)$. g_T is called the infinitesimal generator of T.

Let $\mathcal{E}^1(\underline{\underline{R}}_+,E)$ denote the space of all continuously-differentiable functions from $\underline{\underline{R}}_+$ into E [12]. A linear differential equation on E is an equation of the form

$$Du(t) = A(u(t)) \ ,$$

where $A \in L(E)$ and D is the differentiation operator. Let $e \in E$. A solution to this equation with initial condition e is an $f \in \mathcal{E}^1(\underline{\underline{R}}_+,E)$ with $f(0) = e$, $Df(t) = A(f(t))$ for each $t \in \underline{\underline{R}}_+$. d.s.g.'s are important for the following reason.

THEOREM 3.1 Let E be a l.c.s., and let T be a d.s.g. on E. (a) $Du(t) = g_T(u(t))$ has $t \mapsto T(t)e$ as its unique solution for initial condition $u(0) = e$. (b) g_T uniquely determines T.

The above theorem says that the system (*) is meaningful provided that f is the infinitesimal generator of some d.s.g. on Q. This motivates the following definition. The full subcategory of $\mathrm{Dyn}(K)$ whose objects are the pairs (Q,f) with f the infinitesimal generator of a d.s.g. on Q is called the category of smooth dynamics in K and is denoted $\text{S-Dyn}(K)$. A smooth decomposable system in K is a decomposable system $M = (Q,f,I,g,Y,h)$ in K such that (Q,f) is a smooth dynamics in K. In general, not all dynamics are smooth, so not every decomposable system in K has meaning as a continuous-time system. From now on, the word system shall mean smooth decomposable system, unless stated otherwise.

Some important examples of smooth dynamics are now given.

EXAMPLE 3.2 Let E be any l.c.s., and let $\mathcal{E}_s(\underline{\underline{R}}_+,E)$ be the space of all infinitely-differentiable functions $\underline{\underline{R}}_+ \to E$ with the topology of pointwise convergence of all derivatives. Let D denote the differentiation operator on this space. Then $(\mathcal{E}_s(\underline{\underline{R}}_+,E),D)$ is a smooth dynamics.

EXAMPLE 3.3 $\mathcal{E}(\underline{\underline{R}}_+,E)$ is the same space as above, but with the topology of compact convergence of all derivatives. $(\mathcal{E}(\underline{\underline{R}}_+,E),D)$ is also a smooth dynamics.

EXAMPLE 3.4 $\mathcal{E}'(\underline{\underline{R}}_+)$ is the strong dual [12] of

$\mathcal{E}(\mathbb{R}_+) = \mathcal{E}(\mathbb{R}_+, \mathbb{K})$. It consists of all distributions on $\mathbb{R}_+$ with compact support. Let D be the generalized differentiation operator on this space. Then $(\mathcal{E}'(\mathbb{R}_+), D)$ is a smooth dynamics.

EXAMPLE 3.5 Let $\Delta(\mathbb{R}_+)$ denote the subspace of $\mathcal{E}'(\mathbb{R}_+)$ consisting of those distributions with finite support. They consist of finite linear combinations of the Dirac impulses δ_t at $t \geq 0$ and their derivatives. The operator D on $\mathcal{E}'(\mathbb{R}_+)$ clearly maps $\Delta(\mathbb{R}_+)$ into itself. $(\Delta(\mathbb{R}_+), D)$ is a smooth dynamics.

EXAMPLE 3.6 Let E be a l.c.s. Let $\Delta(\mathbb{R}_+) \otimes_s E$ denote the tensor product of $\Delta(\mathbb{R}_+)$ and E with the strongest locally-convex topology making the canonical map p: $\Delta(\mathbb{R}_+) \times E \to \Delta(\mathbb{R}_+) \otimes E$ separately continuous. $\Delta(\mathbb{R}_+) \otimes E$ may be identified with the space of all E-valued distributions with finite support. $(\Delta(\mathbb{R}_+) \otimes_s E, D \otimes 1_E)$ is a smooth dynamics.

The key to the whole approach here is that given a subcategory $\mathcal{K}$ of LCS, the category S-Dyn($\mathcal{K}$) is isomorphic as an LCS category to an LCS functor category. (S-Dyn($\mathcal{K}$) is regarded as an LCS category by endowing its morphism classes with the LCS structure inherited from $\mathcal{K}$.)

Regard $\Delta(\mathbb{R}_+)$ as a one-object LCS category. The morphisms of $\Delta(\mathbb{R}_+)$ are just its elements. Morphism composition is convolution, and δ_0 is the identity. The following is the key theorem of this paper.

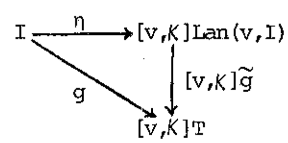
THEOREM 3.7 Let $\mathcal{K}$ be a subcategory of LCS. The LCS category S-Dyn($\mathcal{K}$) is canonically isomorphic to the LCS functor category $[\Delta(\mathbb{R}_+), \mathcal{K}]$. The identification is given by i: $(Q,f) \mapsto T^{\#}$, where $T^{\#}(D^p\delta_t) = f^p \circ T(t)$ with T the unique d.s.g. determined by f. Note that $f = i(D\delta_0)$.

This approach bears some similarity to the approach of Bainbridge [3] to ordinary automata. He let the free monoid X* of a set X be a one-object category, and described dynamics of machines in $\mathcal{K}$ by $[X^*, \mathcal{K}]$ (ordinary functor category).

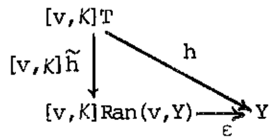
4. Canonical Behavior of Systems

Given a system M = (Q,f,I,g,Y,h) in a subcategory $\mathcal{K}$ of LCS, the behavior of M is its input-output specification. Construction of a behavior requires the construction of natural spaces of input and output signals over time. Unlike the discrete-time case, where these spaces have only one natural structure and have been known for many years, the nature of the inputs and outputs over time in the continuous-time case is not so obvious, and many different structures have been used. In this section, it is demonstrated that relative to the concept of d.s.g, there are natural inputs and outputs over time. This construction necessarily yields canonical reachability and observability maps for M.

The field $\mathbb{K}$ may be regarded as a one-object LCS category whose morphism set is the elements of $\mathbb{K}$. Morphism composition is multiplication; $1 \in \mathbb{K}$ is the identity. With this observation, note that $\mathcal{K}$ may be naturally identified with the LCS functor category $[\mathbb{K}, \mathcal{K}]$. The unique inclusion functor v: $\mathbb{K} \to \Delta(\mathbb{R}_+)$ $k \mapsto k\cdot\delta_0$ induces an LCS functor $[v,\mathcal{K}]$: $[\Delta(\mathbb{R}_+), \mathcal{K}] \to [\mathbb{K}, \mathcal{K}]$ $T \mapsto T\cdot v$. $[v,\mathcal{K}]$ is just the forgetful LCS functor which takes a dynamics (Q,f) to its underlying space Q. Given a $\mathcal{K} = [\mathbb{K}, \mathcal{K}]$ object I; an LCS-left Kan extension of v along I is an LCS functor $\mathrm{Lan}(v,I) \in [\Delta(\mathbb{R}_+), \mathcal{K}]$, together with a natural transformation η: $I \to [v,\mathcal{K}]\mathrm{Lan}(v,I)$, such that for any other $T \in [\Delta(\mathbb{R}_+), \mathcal{K}]$, g: $I \to [v,\mathcal{K}]T$, there is a unique $\tilde{g}$: $\mathrm{Lan}(v,I) \to T$ such that



commutes [5]. Dually, given a $\mathcal{K}$ object Y, an LCS-right Kan extension of v along Y is an LCS functor $\mathrm{Ran}(v,Y) \to [\Delta(\mathbb{R}_+), \mathcal{K}]$, together with an LCS-natural transformation ε: $[v,\mathcal{K}]\mathrm{Ran}(v,Y) \to Y$ such that for any other $T \in [\Delta(\mathbb{R}_+), \mathcal{K}]$ and h: $[v,\mathcal{K}]T \to Y$, there is a unique $\tilde{h}$: $T \to \mathrm{Ran}(v,Y)$ such that



commutes [5]. K is called behavioral if Lan(v,I) and Ran(v,Y) exist for all pairs of K objects (I,Y).

Given a system $M = (Q,f,I,g,Y,h)$ in the behavioral category K, it is convenient to rewrite it as $I \overset{g}{\to} [v,K](Q \overset{f}{\to} Q) \overset{h}{\to} Y$. Using this notation, the behavior B_M of M is defined to be $\mathrm{Lan}(v,I) \overset{\tilde{g}}{\to} (Q,f) \overset{\tilde{h}}{\to} \mathrm{Ran}(v,Y)$. $\bar{g} = [v,K]g$ is called the reachability map of M and $\bar{h} = [v,K]h$ is called the observability map of M.

EXAMPLE 4.1 Let $K = \underline{\underline{LCS}}$, and let $M = (Q,f,I,g,Y,h)$ be a system in K. $\mathrm{Lan}(v,I) = [\Delta(\underline{\underline{R}}_+)\otimes_s I, D\otimes 1)$ (see Example 3.6). $\eta: I \to \Delta(\underline{\underline{R}}_+)\otimes_s I$ is given by $i \mapsto \delta_0 \otimes i$. The reachability map $\bar{g}: \Delta(\underline{\underline{R}}_+)\otimes_s I \to Q$ is given by $D^p\delta_t \otimes i \mapsto f^p T(t)g(i)$, where T is the d.s.g. determined by f. That is, regard the input $D^p\delta_t\otimes i$ as an impluse input at time -t differentiated p times and of weight i. The response due to this input at time 0, is just $f^p T(t)g(i)$. That is, it is the response at -t, which is $f^p g(i)$, decayed for t time units via the natural response T. The response due to a finite linear combination of inputs is found by superposition.

$\mathrm{Ran}(v,Y) = (\mathcal{E}_s(\underline{\underline{R}}_+,Y),D)$ (see Example 3.2) ε: $\mathcal{E}_s(\underline{\underline{R}}_+,Y) \to Y$ is just evaluation at 0, i.e. $\varepsilon(\phi) = \phi(0)$. The observability map $\bar{h}$ takes a state q to its natural response $t \mapsto T(t)q$.

EXAMPLE 4.2 Let $K = \underline{\underline{WS}}$, the category of all weakly-topologized l.c.s.'s. This category is also behavioral, as guaranteed by the following theorem.

THEOREM 4.3 Let (Q,f) be a smooth system dynamics. Then (Q_s,f) is also a smooth system dynamics, where Q_s is Q with its weak topology.

Thus, $\underline{\underline{WS}}$ is shown to be behavioral by merely converting all topologies to weak topologies in Example 4.1. Note that $\mathcal{E}_s(\underline{\underline{R}}_+,Y)$ must already be carrying its weak topology when Y is.

REMARK: In the entire construction above, $\Delta(\underline{\underline{R}}_+)$ may be replaced by $\mathcal{E}'(\underline{\underline{R}}_+)$. This cuts down the number of d.s.g.'s under consideration, since dynamics now will correspond to $\mathcal{E}'(\underline{\underline{R}}_+)$ modules whose actions are separately continuous. The inputs become richer as $\mathcal{E}'(\underline{\underline{R}}_+)\otimes_s I$, while the outputs stay the same, with only a retopologization. This provides some connection with the $\mathcal{E}'(\underline{\underline{R}}_+)$-module approach of other authors [4], [9], and [10]. A more complete comparison will appear in [8].

5. Canonical Realization of Systems

In the classical case of finite-dimensional linear systems, a system $M = (Q,f,I,g,Y,h)$ is reachable if every state can be achieved by the application of some input, i.e., if its reachability map $\bar{g}$ is surjective. Dually, M is observable if any two states can be distinguished by observing the output, i.e., if its observability map $\bar{h}$ is injective. The generalization of these concepts to systems in a category rests upon the concept of image-factorization system (IFS) [2]. Not surprisingly, this concept generalizes to the concept of image-factorization system relative to $\underline{\underline{LCS}}$.

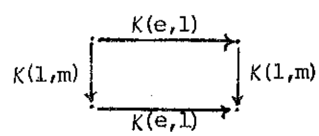
Let K be an $\underline{\underline{LCS}}$ category. An $\underline{\underline{LCS}}$ image-factorization system (LIFS) for K is a pair $(\underline{\underline{E}},\underline{\underline{M}})$ where $\underline{\underline{E}}$ and $\underline{\underline{M}}$ are classes of K morphisms such that

(a) $\underline{\underline{E}}$ and $\underline{\underline{M}}$ are closed under composition;

(b) $e \in \underline{\underline{E}} \Rightarrow e$ is an epimorphism;
$m \in \underline{\underline{M}} \Rightarrow m$ is a monomorphism;

(c) i is an isomorphism $\Rightarrow i \in \underline{\underline{E}} \cap \underline{\underline{M}}$;

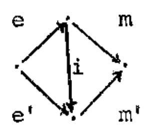
(d) For every $e \in \underline{\underline{E}}$ and $m \in \underline{\underline{M}}$, the diagram



is a pullback in $\underline{\underline{LCS}}$. ($K(e,1)$ is composition with e on the right, i.e., $f \mapsto f\circ e$; $K(1,m)$ is composition with m on the left, i.e., $f \mapsto m\circ f$.)

These conditions differ form the usual requirements of an IFS in K only in condition (d). The IFS condition is:

(d') Every K morphism f has a unique factorization $m \circ e$ with $e \in \underline{\underline{E}}$ and $m \in \underline{\underline{M}}$ which is unique up to isomorphism in the sense that if $m' \circ e'$ is another such factorization, then there is a unique isomorphism i such that



commutes.

THEOREM 5.1 Condition (d) $\Rightarrow$ Condition (d') always. If K is a subcategory of $\underline{\underline{LCS}}$ and $\underline{\underline{E}} \subset$ surjections or $\underline{\underline{M}} \subset$ embeddings, then (d') $\Rightarrow$ (d).

Now assume that K is behavioral, that $M = (Q,f,I,g,Y,h)$ is a system in K, and that $(\underline{\underline{E}},\underline{\underline{M}})$ is a LIFS for K. M is $\underline{\underline{E}}$-reachable if its reachability map $\bar{g} \in \underline{\underline{E}}$, and $\underline{\underline{M}}$-observable if its observability map $\bar{h} \in \underline{\underline{M}}$. M is $(\underline{\underline{E}},\underline{\underline{M}})$-canonical if it is both $\underline{\underline{E}}$-reachable and $\underline{\underline{M}}$-observable.

Let K and $(\underline{\underline{E}},\underline{\underline{M}})$ be as above. A behavior in K is a dynamics of the form $B: \mathrm{Lan}(v,I) \to \mathrm{Ran}(v,Y)$ for some pair of K objects (I,Y). A realization of B is a system $M = (Q,f,I,g,Y,h)$ such that $B = B_M$. The realization problem is to find an $(\underline{\underline{E}},\underline{\underline{M}})$-canonical realization for each behavior.

Finding canonical realizations requires doing factorizations in $[\Delta(\underline{\underline{R}}_+),K]$. That is, B needs to be factored as $\mathrm{Lan}(v,I) \overset{\bar{e}}{\to} (Q,f) \overset{\bar{m}}{\to} \mathrm{Ran}(v,I)$, where $\bar{e}$ (resp. $\bar{m}$) is a "lifting" of a K morphism $e \in \underline{\underline{E}}$ (resp. $m \in \underline{\underline{M}}$) to S-Dyn($K$). Formally, $(\underline{\underline{E}},\underline{\underline{M}})$ lifts to S-Dyn(K) if $(\bar{\underline{\underline{E}}},\bar{\underline{\underline{M}}})$ is an LIFS for K, where $\bar{\underline{\underline{E}}}$ (resp. $\bar{\underline{\underline{M}}}$) is the class of all dynamorphisms in S-Dyn(K) whose underlying K morphism is in $\underline{\underline{E}}$ (resp. $\underline{\underline{M}}$).

LEMMA 5.2 For any subcategory K of $\underline{\underline{LCS}}$ and LIFS $(\underline{\underline{E}},\underline{\underline{M}})$ for K, $(\underline{\underline{E}},\underline{\underline{M}})$ lifts to S-Dyn(K).

Note that $(\underline{\underline{E}},\underline{\underline{M}})$ lifts to Dyn(K) by virture of the dynamorphic-image lemma [1, 4.4]. However, extra work is required to show that the lifting is in S-Dyn(K). As a consequence of Lemma 5.2, the following is obtained.

THEOREM 5.3 For any behavioral subcategory K of $\underline{\underline{LCS}}$ and LIFS $(\underline{\underline{E}},\underline{\underline{M}})$ for K, each behavior in K has an $(\underline{\underline{E}},\underline{\underline{M}})$-canonical realization.

EXAMPLE 5.4 Let $K = \underline{\underline{LCS}}$. Each of the pairs (quotient maps, injections), (surjections, embeddings), (dense maps, closed embeddings) is an IFS for K [7]. Hence, in view of Theorem 5.1, they are also LIFS's, and so lift to S-Dyn(K) by virtue of Theorem 5.2. This says that for infinite-dimensional systems, there are several distinct concepts of canonical realization.

EXAMPLE 5.5 Let $K = \underline{\underline{WS}}$. Since quotients and subspaces of weakly-topologized l.c.s's are also weakly-topologized, it follows that all three of the above are also LIFS's for $\underline{\underline{WS}}$, and so lift to S-Dyn($\underline{\underline{WS}}$).

6. Duality

A classical result in finite-dimensional linear system theory is that a system M is reachable if and only if its dual is observable. In this section, this result is generalized. The approach parallels earlier work on discrete-time systems [6].

Let K be an $\underline{\underline{LCS}}$ category. Its opposite category K^{op} may clearly be regarded as an $\underline{\underline{LCS}}$ category, using the structure inherited from K. An $\underline{\underline{LCS}}$-equivalence $': K^{op} \to K$ is called an $\underline{\underline{LCS}}$ duality functor for K. The most important example of duality functor is the following.

EXAMPLE 6.1 Let $K = \underline{\underline{WS}}$. The functor $': \underline{\underline{WS}}^{op} \to \underline{\underline{WS}}$ which sends each l.c.s. E to its weak dual E'_s and each continuous linear map $f: E \to F$ to its transpose $f': F'_s \to E'_s$ is an $\underline{\underline{LCS}}$ duality functor for $\underline{\underline{WS}}$.

Using the concept of $\underline{\underline{LCS}}$ duality functor, it is possible to define naturally the dual of a system with respect to this functor. More precisely, if K is a subcategory of LCS, $' \; K^{op} \to K$ an $\underline{\underline{LCS}}$ duality functor for K, and $M = (Q,f,I,g,Y,h)$ a system in K, the dual of M (with respect to ') is the 6-tuple $M' = (Q',f',Y',g',I',h')$.

THEOREM 6.2 Let K be a behavioral LCS category, $': K^{op} \to K$ an LCS duality functor for K. (a) If M is a system in K, so too is M'. (b) $(M')' = M$, up to isomorphism. (c) If $\bar{g}$ (resp. $\bar{h}$) is the reachability (resp. observability) map for M, then $\bar{g}'$ (resp. $\bar{h}'$) is the observability (resp. reachability) map for M', up to isomorphism.

Thus, the reachability map and observability map are dual concepts in this framework. This paves the way for expressing the duality of the concepts of reachability and observability. All that remains to be done is to characterize the duality of image-factorization systems.

Given any class C of K morphisms, let $\bar{C}$ denote the smallest class of K morphisms which contains C and which is closed under composition and contains all isomorphisms. Given an LCS duality functor $': K^{op} \to K$, let $C' = \overline{\{c' \mid c \in C\}}$.

LEMMA 6.3 Let K be an LCS category, $(\underline{\underline{E}},\underline{\underline{M}})$ an LIFS for K, and $': K^{op} \to K$ an LCS duality. Then $(\underline{\underline{M}}',\underline{\underline{E}}')$ is an IFS for K also, and $((\underline{\underline{M}}')',(\underline{\underline{E}}')') = (\underline{\underline{E}},\underline{\underline{M}})$.

Combining Theorem 6.2 and Lemma 6.3 gives:

THEOREM 6.4 Under the conditions of the above theorem, and assuming that K is behavioral, a system M in K is $\underline{\underline{E}}$-reachable (resp. $\underline{\underline{M}}$-observable, resp. $(\underline{\underline{E}},\underline{\underline{M}})$-canonical) if and only if M' is $\underline{\underline{E}}'$-observable (resp. $\underline{\underline{M}}'$-reachable, resp. $(\underline{\underline{M}}',\underline{\underline{E}}')$-canonical.

EXAMPLE 6.5 Under the duality functor ' of WS, the transformations of the LIFS's of Example 5.5 under ' are as follows:

((injections)',(quotient maps)') = (dense maps, closed embeddings).

((closed embeddings)',(dense maps)') = (quotient maps, injections).

((embeddings)',(surjections)') = (surjections, embeddings).

The important point to notice here is that when going from a system to its dual, the <u>concepts</u> of reachability, observability, and canonicity may change if M is infinite dimensional. Hence, when discussing duality, one concept of canonical realization will not suffice in general.

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