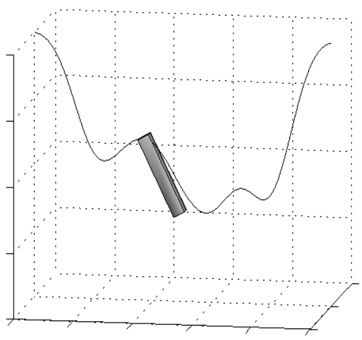
Bone on a wire

Make a simulation of a *bone on a wire*, i.e. a simulaion of a rigid body (the bone) constrained to move with one point along a specific curve (the wire) but otherwise affected by forces (like gravity and air dag) acting on it.



A bone on a wire.

Representation Represent the bone by a rigid block. Use the rigid body structure suggested in [1]

`x q p L`	*State variables*
`R I_inv v omega`	*Derived quantities*
`f tau`	*Computed quantities*
`m I_body I_body_inv`	*Constants*

Include also a body index (or name) if you are to extend the code to several bodies.

The Simulation Loop The simulation loop should roghly have the following structure

```
Initial conditions

while (running)

    Calculate position with respect to wire

    Accumulate external force and torque

    Compute constraint force and torque

    Take a time step

    Calculate the energy of the system

    Visualization and rendering

end
```

Initialization Assign values on constants and choose values for initial position, rotation and linear and angular momentum. Make sure that the initial position is "on" the wire. The visual and physical shape of the bone need not be identical. The inertia tensor for standard geometrical shapes are well known.

The wire See Lab 1.

External force and torque As in Lab 1 the total force (and total torque) is made up by constraint force $\mathbf{f}_c$ (and constraint torque τ_c) and external force (and external torque). Include the gravity force $\mathbf{f}_g = m\mathbf{g}$ (with gravity vector $\mathbf{g}$ acting downwads and with magnitude equal to the gravtational acceleration) and the linear (air) drag force $\mathbf{f}_{air} = -\kappa_{lin}\mathbf{v}$ (with drag coefficient κ_{lin}). The total force on the particle is $\mathbf{f} = \mathbf{f}_g + \mathbf{f}_{air} + \mathbf{f}_c$. The air drag also act to slow down rotation. Include external (air) damping torque $\tau_{air} = -\kappa_{ang}\omega$ (with drag coefficient κ_{ang}). The total torque on the particle is $\tau = \tau_{air} + \tau_c$.

Constraint force and torque The constraint is that a point on the body (with position $\mathbf{x}_p^0$ in the body's local frame of reference) is restricted to move along a line (one particular line segment of the wire). The position of the point in the global frame of reference is $\mathbf{x} = \mathbf{x}_{cm} + \mathbf{x}_p = \mathbf{x}_{cm} + \mathbb{R}\mathbf{x}_p^0$. The constraint is expressed the same way as in Exercise 1 – $0 = c_i \equiv r_{\perp i}$ for $i = 1, 2$ with two orthogonal normals $\mathbf{n}_1$ and $\mathbf{n}_2$. The generalized constraint force (a vector including both force and torque) is $\mathbf{F}_c = \mathbb{J}^T\lambda$. The Lagrangian multiplier is computed from the equation (solve this by computing the inverse of $\mathbb{A}$)

$$\mathbb{A}\lambda = \mathbf{d}$$

with $\mathbb{A} = \mathbb{J}\tilde{\mathbb{M}}^{-1}\mathbb{J}^T$, $\mathbf{d} = -\mathbb{J}\tilde{\mathbb{M}}^{-1}\mathbf{F}_{ext} + \mathbb{J}\tilde{\mathbb{M}}^{-1}\dot{\tilde{\mathbb{M}}}\mathbf{w} - \dot{\mathbb{J}}\mathbf{w}$ and where the Jacobian for this particular constraint and its time derivative are

$$\mathbb{J} = \begin{pmatrix} \mathbf{n}_1^T & (-\mathbf{n}_1 \times \mathbf{x}_p)^T \\ \mathbf{n}_2^T & (-\mathbf{n}_2 \times \mathbf{x}_p)^T \end{pmatrix} \quad , \quad \dot{\mathbb{J}} = \begin{pmatrix} \mathbf{0}^T & (-\mathbf{n}_1 \times [\omega \times \mathbf{x}_p])^T \\ \mathbf{0}^T & (-\mathbf{n}_2 \times [\omega \times \mathbf{x}_p])^T \end{pmatrix}$$

and $\tilde{\mathbb{M}} = \text{diag}(\mathbb{M}, \mathbb{I})$, $\dot{\tilde{\mathbb{M}}} = \text{diag}(0,\ \omega^*\mathbb{I} - \mathbb{I}\omega^*)$ and $\mathbf{w} = [\mathbf{v}, \omega]$. Add spring and damper correcting terms to the Lagrange multipliers $\lambda_i \to \lambda_i - \kappa_s c_i - \kappa_d \dot{c}_i$ to prevent drift.

Time stepping Integrate the equations of motion

$$\dot{Y} = \frac{d}{dt}\begin{pmatrix} \mathbf{x} \\ \mathbf{q} \\ \mathbf{p} \\ \mathbf{L} \end{pmatrix} = \begin{pmatrix} \mathbf{v} \\ \frac{1}{2}[0, \omega] \cdot \mathbf{q} \\ \mathbf{f} \\ \tau \end{pmatrix}$$

using Euler's method and compute the derived quantities $\mathbb{R}$, $\mathbb{I}^{-1}$, $\mathbf{v}$ and ω.

Energy Calculate the total energy of the system. What size of time steps are required for having variations in the total energy no larger than 1% of the average total energy during the simulation, when the air damping is put to zero.

Visualization and rendering Use OSG or glut. See separate documentation.

Comment As in Lab 1, large curvature on the wire can lead to oscillations and energy damping – use a smother curve for the wire in that case.

Bonus It is possible to earn one extra bonus point on this lab. Suggestions: *i)* make analysis of how the total energy depends on size of time step *ii)* incorporate more bodies coupled to the first by constraint *iii)* extend the wire design to arbitrary 3D curves *iv)* implement a mouse-spring for user interaction.

References

[1] A. Witkin, D. Baraff et al *Physically Based Modelling 97 – SIGGRAPH lecture notes.*