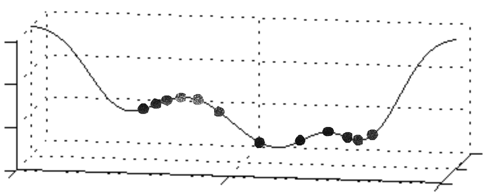
Bead on a wire

Make a simulation of a *bead on a wire*, i.e. a simulation of a particle (the bead) constrained to move along a specific curve (the wire) but otherwise affected by forces (like gravity and air dag) acting on it.



12 beads on a wire.

Representation Represent the bead by a point particle. Use the particle structure suggested in [1]

`x`	*position*
`v`	*velocity*
`f`	*accumulated force*
`m`	*mass*

Include also a particle index (or name) if you are to extend the code to system of particles.

The Simulation Loop The simulation loop should roughly have the following structure

```
Initial conditions

while (running)

    Calculate position with respect to wire

    Accumulate external forces

    Compute constraint force

    Take a time step

    Calculate the energy of the system

    Visualization and rendering

end
```

Initialization Assign values on constants and choose values for initial position and velocity. Make sure that the initial position is on the wire.

The wire Let the wire be represented by N segments of straight lines with start and end points $\{\mathbf{r}_0, \mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_N\}$. The wire can be generated by a function or exported from a drawing tool – design your own wire! The bead-to-wire distance in the direction of the normal $\mathbf{n}_i$ can be computed by

$$r_{\perp,\ i} = (\mathbf{x} - \mathbf{r}_{a-1}) \cdot \mathbf{n}_i$$

$\mathbf{n}_1$ and $\mathbf{n}_2$ are two orthogonal normals to the line segment between $\mathbf{r}_{a-1}$ and $\mathbf{r}_a$ that the particle currently follows. There is a space of normal vectors to chose among – chose any two orthogonal normals. The simulation may be simplified by restricting the wire to be a 2D height map (for instance in the $y = 0$ plane). The tangent and normals to such a wire are $\mathbf{t} = [r_{xn+1} - r_{xn}, 0, r_{zn+1} - r_{zn}]$, $\mathbf{n}_1 = [-t_z, 0, t_x]/|\mathbf{t}|$ and $\mathbf{n}_2 = [0, 1, 0]$.

External forces Besides the constraint force $\mathbf{f}_c$ the total force $\mathbf{f}$ should include two external forces – the gravity force $\mathbf{f}_g = m\mathbf{g}$ (with gravity vector $\mathbf{g}$ acting in negative z-direction and with magnitude equal to the gravitational acceleration) and the air drag force $\mathbf{f}_{air} = -\kappa_{air}\mathbf{v}$ (with drag coefficient κ_{air}) and the constraint force $\mathbf{f}_c$. The total force on the particle is $\mathbf{f} = \mathbf{f}_g + \mathbf{f}_{air} + \mathbf{f}_c$.

Constraint force The constraint force $\mathbf{f}_c$ follows from the constraints $0 = c_i \equiv r_{\perp\, i}$ for $i = 1, 2$ (saying *zero distance between bead and wire*) in the directions $\mathbf{n}_1$ and $\mathbf{n}_2$. This condition implies the constraint force $\mathbf{f}_c = \sum_j \lambda_j \mathbf{n}_j$ with Lagrange multipliers $\lambda_i = -\mathbf{f}_e \cdot \mathbf{n}_i - \kappa_s c_i - \kappa_d \dot{c}_i$, where we include spring and damper correction to the constraint force to prevent the bead from gradually drifting away from the wire. Choose suitable values for the spring and damper coefficients κ_s and κ_d.

Time stepping The motion of the particle is governed by Newtons law of motion $\mathbf{f} = m\mathbf{a}$. The particle velocity and position evolves in time according to $m\dot{\mathbf{v}} = \mathbf{f}$ and $\dot{\mathbf{x}} = \mathbf{v}$. Use Euler's method for integrating these equations.

Energy Calculate the total energy of the system. What size of time steps are required for having variations in the total energy no larger than 1% of the average total energy during the simulation, when the air damping is put to zero.

Visualization and rendering Use OSG or glut. See separate documentation.

Comment 1 The motion of the bead is constrained to the wire using point-to-line constraint. The bead will have a slightly wrong direction of velocity when going from one line segment to the next. This is compensated by the spring-damper correction to the constraint force. Depending on the values of the spring and damper constants, the total energy may oscillate or be damped out. The size of this effect depends on the number of line segments and the amount of curvature of the curve – choose a smoother curve if you are having problems.

Comment 2 There are several ways to extend this lab to arbitrary wire design in 3D. Feel free to experiment!

References

[1] A. Witkin, D. Baraff et al *Physically Based Modelling 97 – SIGGRAPH lecture notes.*