Exempel på icke-linjärt minsta-kvadratproblem, forts.

Då blir jacobianen $J = \begin{bmatrix} J_1 \\ \vdots \\ J_m \end{bmatrix}$,

$$J_i = \begin{bmatrix} J_{i1} & J_{i2} & J_{i3} \end{bmatrix},$$

$$J_{i1} = \begin{bmatrix} \frac{\partial f_{ix}}{\partial c_x} & \frac{\partial f_{ix}}{\partial c_y} \\ \frac{\partial f_{iy}}{\partial c_x} & \frac{\partial f_{iy}}{\partial c_y} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$J_{i2} = \begin{bmatrix} \frac{\partial f_{ix}}{\partial a} & \frac{\partial f_{ix}}{\partial b} & \frac{\partial f_{ix}}{\partial \delta} \\ \frac{\partial f_{iy}}{\partial a} & \frac{\partial f_{iy}}{\partial b} & \frac{\partial f_{iy}}{\partial \delta} \end{bmatrix} = \begin{bmatrix} -\cos\delta\cos\theta_i & \sin\delta\sin\theta_i & a\sin\delta\cos\theta_i + b\cos\delta\sin\theta_i \\ -\sin\delta\cos\theta_i & -\cos\delta\sin\theta_i & -a\cos\delta\cos\theta_i + b\sin\delta\sin\theta_i \end{bmatrix}$$

$$J_{i3} = \begin{bmatrix} \frac{\partial f_{ix}}{\partial \theta_1} \cdots \frac{\partial f_{ix}}{\partial \theta_{i-1}} & \frac{\partial f_{ix}}{\partial \theta_i} & \frac{\partial f_{ix}}{\partial \theta_{i+1}} \cdots \frac{\partial f_{ix}}{\partial \theta_m} \\ \frac{\partial f_{iy}}{\partial \theta_1} \cdots \frac{\partial f_{iy}}{\partial \theta_{i-1}} & \frac{\partial f_{iy}}{\partial \theta_i} & \frac{\partial f_{iy}}{\partial \theta_{i+1}} \cdots \frac{\partial f_{iy}}{\partial \theta_m} \end{bmatrix} = \begin{bmatrix} 0 \ldots 0 & a\cos\delta\sin\theta_i + b\sin\delta\cos\theta_i & 0 \ldots 0 \\ 0 \ldots 0 & a\sin\delta\sin\theta_i - b\cos\delta\cos\theta_i & 0 \ldots 0 \end{bmatrix}$$

Exempel på icke-linjärt minsta-kvadratproblem, forts.

Andraderivatorna:

$\nabla^2 f_{ix} =$

$$
\begin{bmatrix}
 & c_x & c_y & a & b & \delta & & \theta_i & \\
c_x & 0 & 0 & 0 & 0 & 0 & 0\ldots0 & 0 & 0\ldots0 \\
c_y & 0 & 0 & 0 & 0 & 0 & 0\ldots0 & 0 & 0\ldots0 \\
a & 0 & 0 & 0 & 0 & \sin\delta\cos\theta_i & 0\ldots0 & \cos\delta\sin\theta_i & 0\ldots0 \\
b & 0 & 0 & 0 & 0 & \cos\delta\sin\theta_i & 0\ldots0 & \sin\delta\cos\theta_i & 0\ldots0 \\
\delta & 0 & 0 & \sin\delta\cos\theta_i & \cos\delta\sin\theta_i & a\cos\delta\cos\theta_i - b\sin\delta\sin\theta_i & 0\ldots0 & -a\sin\delta\sin\theta_i + b\cos\delta\cos\theta_i & 0\ldots0 \\
 & 0 & 0 & 0 & 0 & 0 & 0\ldots0 & 0 & 0\ldots0 \\
 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 & 0 & 0 & 0 & 0 & 0 & 0\ldots0 & 0 & 0\ldots0 \\
\theta_i & 0 & 0 & \cos\delta\sin\theta_i & \sin\delta\cos\theta_i & -a\sin\delta\sin\theta_i + b\cos\delta\cos\theta_i & 0\ldots0 & a\cos\delta\cos\theta_i - b\sin\delta\sin\theta_i & 0\ldots0 \\
 & 0 & 0 & 0 & 0 & 0 & 0\ldots0 & 0 & 0\ldots0 \\
 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 & 0 & 0 & 0 & 0 & 0 & 0\ldots0 & 0 & 0\ldots0
\end{bmatrix}
$$

Exempel på icke-linjärt minsta-kvadratproblem, forts.

$$\nabla^2 f_{iy} =$$

$$\begin{bmatrix}
 & c_x & c_y & a & b & \delta & & \theta_i & \\
c_x & 0 & 0 & 0 & 0 & 0 & 0\ldots0 & 0 & 0\ldots0 \\
c_y & 0 & 0 & 0 & 0 & 0 & 0\ldots0 & 0 & 0\ldots0 \\
a & 0 & 0 & 0 & 0 & -\cos\delta\cos\theta_i & 0\ldots0 & \sin\delta\sin\theta_i & 0\ldots0 \\
b & 0 & 0 & 0 & 0 & \sin\delta\sin\theta_i & 0\ldots0 & -\cos\delta\cos\theta_i & 0\ldots0 \\
\delta & 0 & 0 & -\cos\delta\cos\theta_i & \sin\delta\sin\theta_i & a\sin\delta\cos\theta_i + b\cos\delta\sin\theta_i & 0\ldots0 & a\cos\delta\sin\theta_i + b\sin\delta\cos\theta_i & 0\ldots0 \\
 & 0 & 0 & 0 & 0 & 0 & 0\ldots0 & 0 & 0\ldots0 \\
 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 & 0 & 0 & 0 & 0 & 0 & 0\ldots0 & 0 & 0\ldots0 \\
\theta_i & 0 & 0 & \sin\delta\sin\theta_i & -\cos\delta\cos\theta_i & a\cos\delta\sin\theta_i + b\sin\delta\cos\theta_i & 0\ldots0 & a\sin\delta\cos\theta_i + b\cos\delta\sin\theta_i & 0\ldots0 \\
 & 0 & 0 & 0 & 0 & 0 & 0\ldots0 & 0 & 0\ldots0 \\
 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 & 0 & 0 & 0 & 0 & 0 & 0\ldots0 & 0 & 0\ldots0
\end{bmatrix}$$