Overview

- Parametric Bi-Cubic Surfaces
  - Hermite Surfaces
  - Bézier Surfaces
  - B-Spline Surfaces
- Quadric Surfaces
- Surfaces of revolution
- Sweep Representation

Parametric Bi-cubic Surfaces

We have $Q(t) = T \cdot M \cdot G$ from parametric curves.

Replace t with s (need a parameter in the other direction).

$$Q(s) = S \cdot M \cdot G$$

The parametric bi-cubic surface is given by

$$Q(s,t) = S \cdot M \cdot G \cdot M^T \cdot T^T, \qquad 0 \le s,t \le 1$$

where

$$G = \begin{bmatrix} g_{11} & g_{21} & g_{31} & g_{41} \\ g_{12} & g_{22} & g_{32} & g_{42} \\ g_{13} & g_{23} & g_{33} & g_{43} \\ g_{14} & g_{24} & g_{34} & g_{44} \end{bmatrix}$$

Hermite Surfaces

Hermite Surface is represented by:

$$x(s,t) = S \cdot M_H \cdot G_{H_x} \cdot M_H^T \cdot T^T$$

$$y(s,t) = S \cdot M_H \cdot G_{H_y} \cdot M_H^T \cdot T^T$$

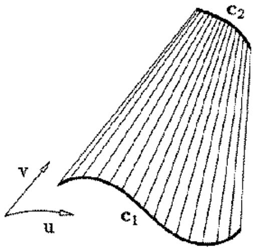
$$z(s,t) = S \cdot M_H \cdot G_{H_z} \cdot M_H^T \cdot T^T$$

Ruled surface:

The four interpolants

$Q(0,t) \quad Q(1,t) \quad Q(s,0) \quad Q(s,1)$

are straight lines.



Hermite Surfaces

G_{H_x} can be written as:

$$G_{H_x} = \begin{bmatrix} g_{11} & g_{21} & g_{31} & g_{41} \\ g_{12} & g_{22} & g_{32} & g_{42} \\ g_{13} & g_{23} & g_{33} & g_{43} \\ g_{14} & g_{24} & g_{34} & g_{44} \end{bmatrix} = \begin{bmatrix} x(0,0) & x(0,1) & \frac{\partial}{\partial t}(0,0) & \frac{\partial}{\partial t}(0,1) \\ x(1,0) & x(1,1) & \frac{\partial}{\partial t}(1,0) & \frac{\partial}{\partial t}(1,1) \\ \frac{\partial}{\partial s}(0,0) & \frac{\partial}{\partial s}(0,1) & \frac{\partial^2}{\partial t \partial s}(0,0) & \frac{\partial^2}{\partial t \partial s}(0,1) \\ \frac{\partial}{\partial s}(1,0) & \frac{\partial}{\partial s}(1,1) & \frac{\partial^2}{\partial t \partial s}(1,0) & \frac{\partial^2}{\partial t \partial s}(1,1) \end{bmatrix}$$

Upper left 2x2 matrix contains the x coordinate of the four corners.

Upper right and lower left 2x2 matrix give coordinates for the tangent vector along each parametric direction.

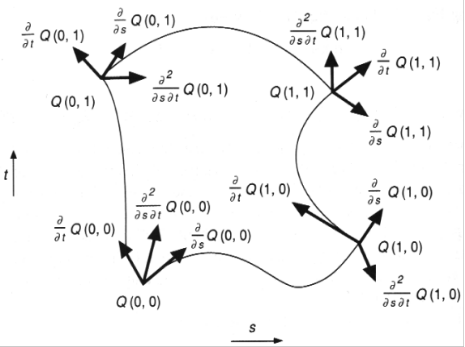
Lower right 2x2 matrix is the partial derivatives with respect to both s and t. (Called twist).

Hermite Surfaces

Hermite permits C^1 and G^1 continuity from one curve segment to the next.

Conditions for C^1 continuity are that the control points along the edge, the tangent vectors and the twist vectors to be equal.

For G^1 the vectors must have the same direction (not the same magnitude).



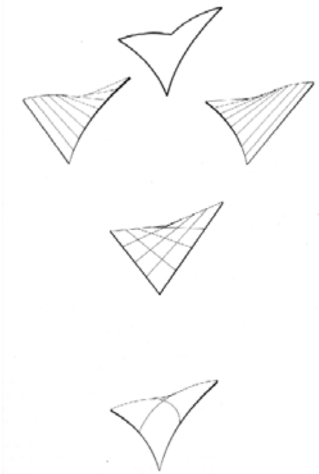
Coons Patch

Hermite surfaces are a restricted form of Coons patch.

Consists of two ruled surfaces and one bilinear blend.

$$x = r_u + r_v - r_{uv}$$

ruled + ruled - bilinear



Bézier Surfaces

Derived the same way as the Hermite:

$$x(s,t) = S \cdot M_B \cdot G_{B_x} \cdot M_B^T \cdot T^T$$

$$y(s,t) = S \cdot M_B \cdot G_{B_y} \cdot M_B^T \cdot T^T$$

$$z(s,t) = S \cdot M_B \cdot G_{B_z} \cdot M_B^T \cdot T^T$$

The geometry matrix G consists of 16 control points.

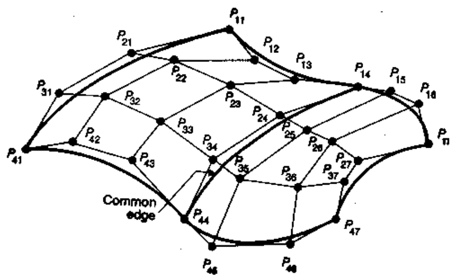
Bézier surfaces are attractive in interactive design.

- The surface interpolates some control points
- Convex hull
- Subdivision

Bézier Surfaces

C^0 and G^0 is created by making the four common control points equal.

G^1 occurs when two sets of four control points on either side of the edge are collinear with the points on the edge.



Bézier Surfaces

A Bézier surface can be displayed using the following equation:

$$x(s,t) = \begin{bmatrix}(1-t)^3 & 3t(1-t)^2 & 3t^2(1-t) & t^3\end{bmatrix} \cdot G_{B_x} \cdot \begin{bmatrix}(1-s)^3 \\ 3s(1-s)^2 \\ 3s^2(1-s) \\ s^3\end{bmatrix}$$

B-Spline Surfaces

B-spline surfaces are represented as:

$$x(s,t) = S \cdot M_{Bs} \cdot G_{Bs_x} \cdot M_{Bs}^T \cdot T^T$$

$$y(s,t) = S \cdot M_{Bs} \cdot G_{Bs_y} \cdot M_{Bs}^T \cdot T^T$$

$$z(s,t) = S \cdot M_{Bs} \cdot G_{Bs_z} \cdot M_{Bs}^T \cdot T^T$$

B-spline surfaces come naturally with C^2 continuity.

All techniques for subdivision and display carry over to the bi-cubic case.

Surface Normal

The normal to a bi-cubic surface is needed for shading.

The s tangent vectors and the t tangent vectors are both parallel to the surface at the point (s,t) and their cross product is perpendicular to the surface.

The normal can then be calculated:

$$\frac{\partial}{\partial s}Q(s,t) \times \frac{\partial}{\partial t}Q(s,t) = \begin{bmatrix} y_s z_t - y_t z_s & z_s x_t - z_t x_s & x_s y_t - x_t y_s \end{bmatrix}$$

Quadric Surfaces

A frequently used class of objects, which are described with second-degree equations.

The implicit surface equation

$$f(x,y,z) = ax^2 + by^2 + cz^2 + 2dxy + 2eyz + 2fxz + 2gx + 2hy + 2jz + k = 0$$

defines a family of quadric surfaces. An alternative to rational surfaces if only quadric surfaces are being represented.

Useful in specialized applications, such as molecular modeling.

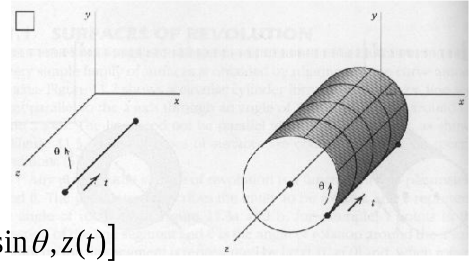
Other objects are spheres, ellipsoids, tori, parapoloids and hyperbolas.

Surface of Revolution

A very simple way of defining objects is obtained by rotating a curve or a line around an axis.

A circular cylinder is formed by rotating a line segment parallel to the z-axis through an angle of 360° around the same axis.

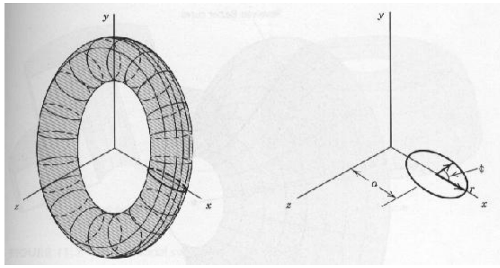
Any point on the surface of revolution is a function of t and θ.



$$P(t,\theta) = [x(t)\cos\theta, x(t)\sin\theta, z(t)]$$

Surface of Revolution

A simple torus is generated by rotating a circle in the xz-plane around the z-axis.

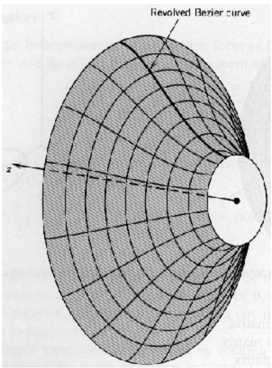


The torus is represented by:

$$P(\phi,\theta) = [(a + r\cos\phi)\cos\theta, (a + r\cos\phi)\sin\theta, a\sin\phi]$$

Surface of Revolution

Surfaces of revolution can also be obtained by rotating a curve around an axis.

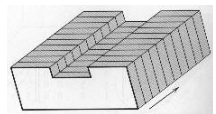


Sweep Representation

A technique to produce 3D objects.

Specify a 2D shape and sweep the shape through a region.

The sweep transformation can contain translation, scaling or rotation.



Translational Sweep