Overview Lecture 4

- Projections
  - Parallel
  - Perspective
- 3D View Volume
- 3D Viewing Transformation
- Camera Model - Assignment 2
- OFF files

Projections - 3D Viewing

- 3D more complex than 2D
  - One more dimension
  - Display device still 2D
- Analog to taking a photograph

Projection - 3D viewing

Parallel Projection

- Orthographic
  - Top
  - Front
  - Side
  - Axonometrix
    - Isometric
- Oblique
  - Cabinet
  - Cavalier

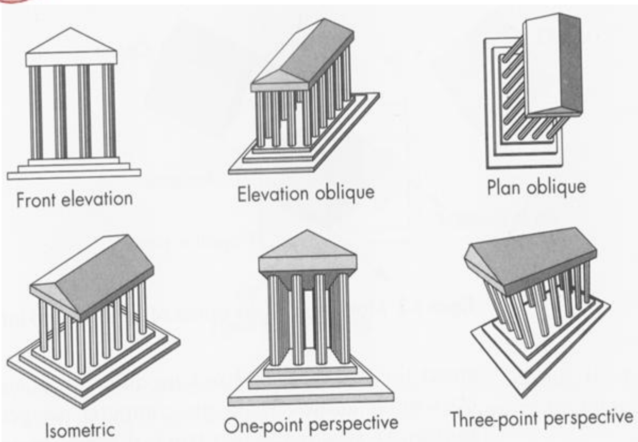
Perspective Projection

- One point
- Two point
- Three point
- Camera model

Projections

- Determined by where you place the projection plane relative to principal axes, and what angle the projectors make with the projection plane.
- Parallel projections are used in engineering and architecture drawings, because they can be used for measurements.
- Perspective projection imitates our eyes or camera and looks more natural.

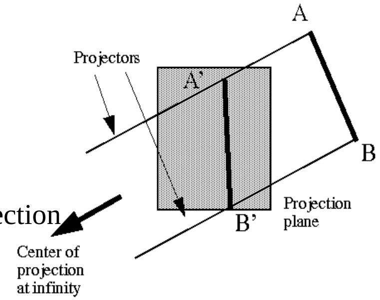
Projections



Projection - 3D viewing Parallel Projection

If object positions are transformed to the projection plane along parallel lines.

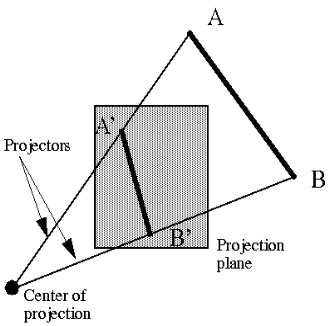
Determined by Direction of Projection



Projection - 3D viewing Perspective Projection

If object positions are transformed to the projection plane along lines that converge to **center of projection**

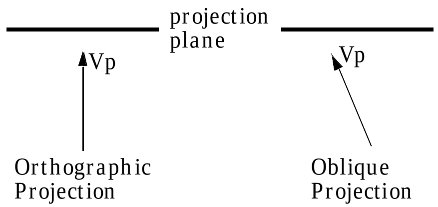
or **prp**.



prp
projection reference point

Projection - 3D viewing Parallel Projection

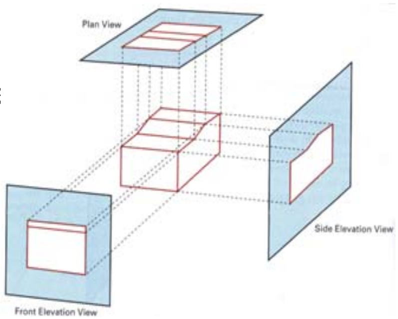
- Preserve relative proportions of objects.
- **Orthographic:**
  - Direction of projection is normal to the projection plane.
- **Oblique:**
  - The projection plane and the direction of projection are not perpendicular to each other.



Projection - 3D viewing
Parallel Orthographic Projection

- **Top (Plan View)**
- **Front Elevation**
- **Side Elevation**
- **Rear Elevation**
- 3D nature difficult to see
- Commonly used in engineering and architectural drawings.

- Length and angles can be measured accurately from the drawings.

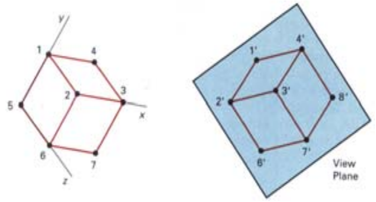


Projection - 3D viewing
Axonometric Orthographic

- Can display more than one face of an object.
- The projection plane is not normal to a principal axis.
- Uniform foreshortening. More like perspective.
- Parallelism of lines are preserved, but not angles.
- *Isometric, dimetric, trimetric.*

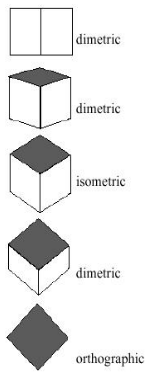
Projection - 3D viewing
Isometric Orthographic

- The projection plan intersects each coordinate axis at the same distance.
- The projection plane makes equal angles (120°) with each principal axis.
  - Allowing measurements along the axes to be made to the same scale.

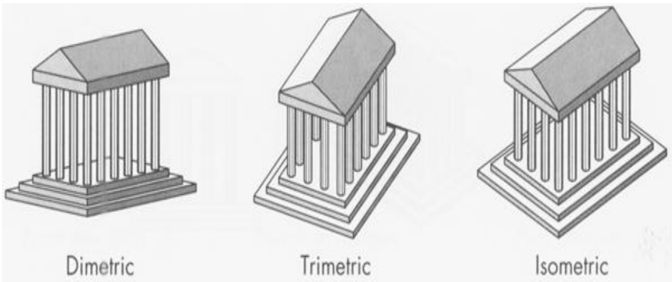


Projection - 3D viewing
Dimetric and Trimetric Orthographic

- **Dimetric**: Angles between two of the principal axes are equal.
- Need two scale ratios.
- **Trimetric**: Angles different between the three principal axes.
- Need three scale ratios.



Dimetric, Trimetric and Isometric

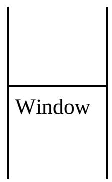


Projection - 3D viewing Parallel Oblique Projection

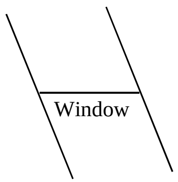
- Direction of projection is not normal to the projection plane.
- The projection plane is normal to a principal axis, so the projection of the face of the object parallel to this plane allows measurement of angles and distances.
- Other faces allow the measurement of distances along principal axes, but not angles.

Oblique vs. Orthographic

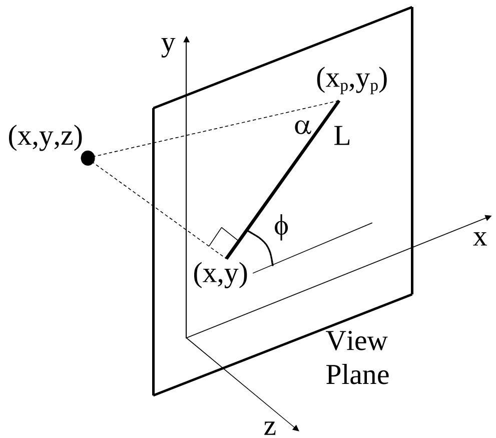
Orthographic Projection



Oblique Projection



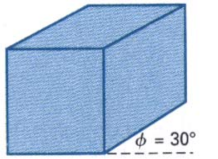
Parallel Oblique Projection



Projection - 3D viewing Parallel Oblique Projection

Cavalier:

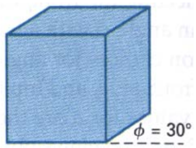
- The direction of projection makes 45° angle with the projection plane.
- Depth = width and height.



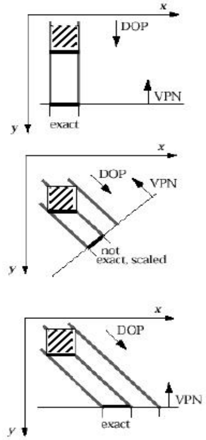
No foreshortening

Cabinet:

- The direction of projection makes an angle of arctan(2) = 63,4° with the projection plane.
- Foreshortening of a half → more 3D realistic.



Summary of Parallel Projections



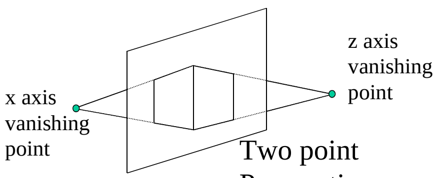
1) Multiview Orthographic
- VPN ∥ a principal coordinate axis
- DOP ∥ VPN
- shows single face, exact measurements

2) Axonometric
- VPN ∦ a principal coordinate axis
- DOP ∥ VPN
- adjacent faces, none exact, uniformly foreshortened (as a function of angle between face normal and DOP)

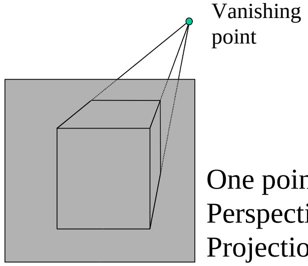
3) Oblique
- VPN ∥ a principal coordinate axis
- DOP ∦ VPN
- adjacent faces, one exact, others uniformly foreshortened

Projection - 3D viewing Perspective Projection

Perspective projection is categorized by their number of vanishing points and therefore by the number of axes the projection plane cuts.
Do not preserve relative proportions of objects.

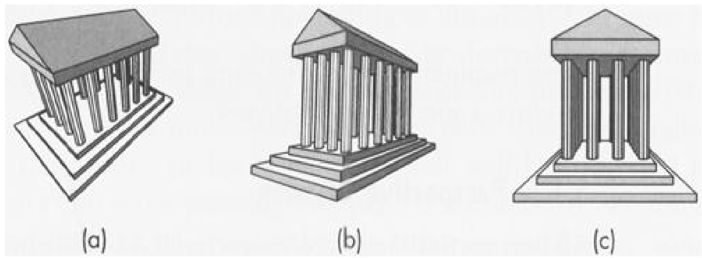


Two point Perspective Projection



One point Perspective Projection

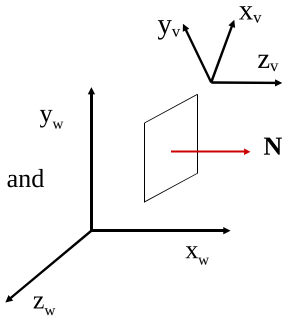
Perspective Projections



3 point   2 point   1 point

3D Viewing Coordinate System

- **View Reference Point**
  - Origin.
- **View-Plane Normal N**
  - Positive direction for the viewing z_v axis and
  - the orientation of the view plane.
- **View-Up Vector V**
  - The up direction for the view.
  - Positive direction for y_v. V not parallel to N.
- **U Vector**
  - Perpendicular to both V and N.

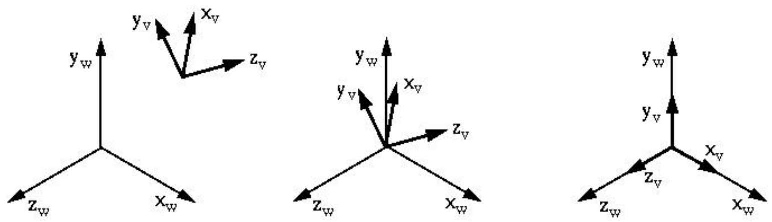


3D Viewing Transformation

Viewing Pipeline:

World Coord. ⇨ **Viewing Coord**. ⇨ Device Coord.

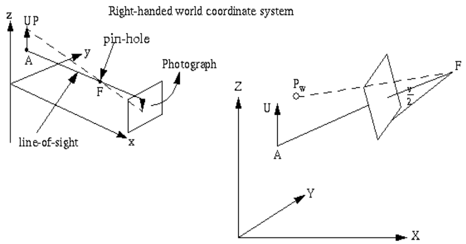
Need to establish a view reference coordinate system.



Camera Model

From Point - F:	The position of the camera.
At Point - A:	Where the camera is aimed.
Up vector - U:	Defines the up direction.
View angle - v:	

Field of view.

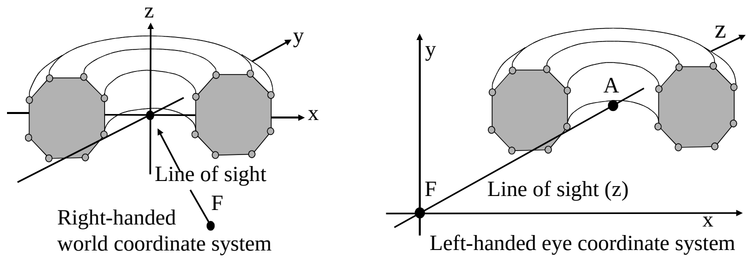


Viewing Transformation

- Transform 3D world coordinates (x_w, y_w, z_w) into 3D eye coordinates (x_e, y_e, z_e).
- Transform 3D eye coordinates (x_e, y_e, z_e) into 2D normal device coordinates (x_{ndc}, y_{ndc}).

- **F** ends up in the origin of the eye coordinate system. **A** ends up on the positive z-axis. **U**P vector ends up in the positive Y-Z plane.

Camera Model



$$P_e = (P_w - F)V$$

World to Eye Transformation

Line-of-sight vector

$$c = \frac{A-F}{\|A-F\|}$$

Z-axis of eye coord. should be mapped to $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

World to Eye Transformation

Vector perpendicular to U and A-F

$$a = \frac{(A-F)\times U}{\|(A-F)\times U\|}$$

X-axis of eye coord. should be mapped to $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

World to Eye Transformation

Vector perpendicular to *c* and *a*

$$b = \frac{((A-F)\times U)\times(A-F)}{\|((A-F)\times U)\times(A-F)\|}$$

Y-axis of eye coord. should be mapped to $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

World to Eye Transformation

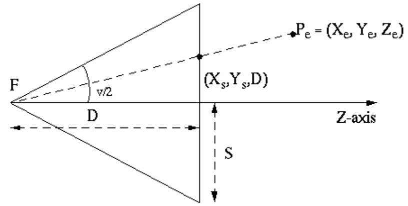
Combining all three conditions

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

V is orthogonal, so

$$V^t = V^{-1}$$

Convert From Eye to NDC 3D to 2D



$$\tan\left(\frac{v}{2}\right) = \frac{S}{D}$$

$$\frac{x_s}{D} = \frac{x_e}{z_e} \qquad \frac{y_s}{D} = \frac{y_e}{z_e}$$

Convert From Eye to NDC 3D to 2D

$$x_{ndc} = x_{Vc} + \frac{\left(\frac{x_e}{z_e} \cdot V_{width}\right)}{2 \cdot \tan\left(\frac{v}{2}\right)}$$

$$y_{ndc} = y_{Vc} + \frac{\left(\frac{y_e}{z_e} \cdot V_{height}\right)}{2 \cdot \tan\left(\frac{v}{2}\right)}$$

Restriction on F, A, U, v

- F and A may not be the same point
  - not able to define line-of-sight
- U cannot be a null vector
  - need a unique **up** direction
- U cannot be parallel to line-of-sight
  - need a unique rotational position
- v must be $0° < \mathbf{v} < 180°$

Zoom

- Enlarge an image by reducing the angle *v*.
- Increasing the view angle makes the image smaller.
- Viewing angles between 40° and 60° give the most realistic view.

OFF File Format

OFF #header

Nvertices Nfaces Nedges

X[0] Y[0] Z[0]

: : :

X[Nv-1] Y[Nv-1] Z[Nv-1]

NV V[0] V[1] …..V[NV-1] COLORSPEC

We will not use COLORSPEC, read and discard.

OFF File Format Example

cube.off

```
OFF
8 6 24
0 0 0
0 0 1
0 1 0
0 1 1
1 0 0
1 0 1
1 1 0
1 1 1
4 0 1 3 2
4 2 3 7 6
4 4 6 7 5
4 0 4 5 1
4 1 5 7 3
4 0 2 6 4
```