TDBB08   Logic with Applications   Spring 1998b

Obligatory Exercise 4
Due date: May 20, 1998 at 1500

Note: These exercises are to be turned in to Petter Edblom. Do not place them in the mail shelf of Stephen Hegner, or you will risk lateness penalties. All solutions of obligatory exercises must be completed individually. It is not permitted to copy, in whole or in part, the solutions of another and submit those solutions as one's own. Neither is it permitted to develop solutions within groups working together. Discussion of general course concepts is, of course, permitted.

1. Using the normalization algorithm of the course notes, convert the following sentence to a set of clauses with no existential quantifiers and no common variables.

$$(\forall x)(\exists w)(P(x,f(w))) \rightarrow (\neg(\exists y)(\forall x)(\forall w)(\neg Q(w) \vee (R(x) \wedge S(f(y),w))))$$

2. Given is the following formula.

$$(\forall x)(S(y) \wedge ((\exists w)P(x,w) \rightarrow Q(x,w)) \wedge (\forall x)(\forall y)R(x,y) \wedge R(x,y))$$

Do the following.

(i)   Draw the parse tree for this formula, using the conventions given in the course notes.
(ii)   Characterize each instance of each variable occurring in a term as free or bound.
(iii)   Rename the variables in this formula, giving an equivalent formula with the property that no variable occurs in more than one quantifier, and no variable is both bound and free. In your solution, only bound variables may be renamed; free variables may not be renamed.
(iv)   Draw the parse tree for your formula of (iii).

Note: The scopes of quantifiers in this formula are deliberately unusual, in order to capture as many situations as possible. Pay careful attention to the locations of the parentheses.