TDBB08 Logic with Applications Spring 1998b

Obligatory Exercise 3
Due date: May 6, 1998 at 1500

Note: These exercises are to be turned in to Petter Edblom. Do not place them in the mail shelf of Stephen Hegner, or you will risk lateness penalties.

All solutions of obligatory exercises must be completed individually. It is not permitted to copy, in whole or in part, the solutions of another and submit those solutions as one's own. Neither is it permitted to develop solutions within groups working together. Discussion of general course concepts is, of course, permitted.

1. Consider the logical operation of Exclusive OR (denoted by $\oplus$) as defined by the following truth table.

φ_1	φ_2	$(\varphi_1 \oplus \varphi_2)$
0	0	0
0	1	1
1	0	1
1	1	0

Give left and right Gentzen-style proof rules for this operation. The rules should be both non-weakening and simplifying. In defining "simplifying" in this case, take the definition of the complexity of Exclusive or to be:

$$\text{Complexity}(\varphi_1 \oplus \varphi_2) = 1 + \text{Complexity}(\varphi_1) + \text{Complexity}(\varphi_2)$$

In addition to providing the rule, explain how and why it works.

2. Give a proof, in the Gentzen sequent system G′, of the following

$$\{(\neg A \vee B), \neg(C \wedge \neg A \wedge \neg B), \neg(D \wedge \neg C), D\} \models (B \vee E)$$

Your initial sequent should contain only these formulas; do **not** do any conversion first (*e.g.,* to CNF or to a refutation formulation).

3. Convert the following rules to an equivalent set of Horn clauses, and use positive unit resolution to compute the set of atomic consequences of the resulting set. Show your answer in the form of a resolution graph.

$\{A_5,$
$A_6,$
$(A_1 \wedge A_2 \wedge A_3 \wedge A_4) \rightarrow (A_9 \wedge A_8 \wedge A_7),$
$(A_5 \wedge A_6) \rightarrow (A_{11} \wedge A_{12}),$
$(A_9 \wedge A_7 \wedge A_{12}) \rightarrow A_1,$
$(A_7 \wedge A_8 \wedge A_{12}) \rightarrow A_9,$
$A_{12} \rightarrow (A_2 \wedge A_3 \wedge A_9 \wedge A_4),$
$(A_3 \wedge A_{11}) \rightarrow A_{10},$
$(A_7 \wedge A_{10}) \rightarrow \perp,$
$(A_1 \wedge A_{11}) \rightarrow \perp\}$

4. Determine whether or not each of the following sets of formulas is equivalent to a set of Horn clauses. In the case that the given set Φ is equivalent to a set of Horn clauses Φ_H, provide such a set Φ_H, together with a justification for the equivalence. If the given set Φ is not equivalent to a set of Horn clauses, give a set of atoms Δ such that $\Phi \cup \Delta$ does not have a least model, and show why no such least model exists.

(a) $\{\neg A \vee B \vee C, \neg B \vee C, \neg C \vee B, A\}$
(b) $\{A \vee B \vee C, \neg B, \neg A \vee C\}$
(c) $\{A \vee B \vee C, \neg B, B\}$
(d) $\{A \vee B \vee C, B \vee C, C\}$
(e) $\{A \vee B, \neg A \vee \neg B\}$