

**Exercises for the group exercise session on  
May 15, 1998**

1. Apply the normalization algorithm (which converts a sentence to a form suitable for resolution) to each of the sentences of Exercise 1 on page 225 of the textbook. Note that these examples do not involve function or constant symbols in the initial formula, although such symbols may be introduced via Skolemization.
2. Apply the normalization algorithm (which converts a sentence to a form suitable for resolution) to the following sentences.

$$\neg(\forall x)(P(x) \rightarrow ((\forall y)(P(y) \rightarrow P(f(x,y))) \wedge \neg(\forall y)(Q(y,x) \rightarrow P(y))))$$

$$(\forall x)(\exists y)(P(x,g(x,y)) \rightarrow (\exists y)(Q(f(x,y,y))))$$

3. Time permitting, work Exercise 9.4 on pages 195-196 and Exercise 9.5 on page 198 of the textbook. Apply all of the normalization algorithm to each sentence.
4. Exhibit a resolution refutation for each of the following sets of clauses. Remember that a, b, c, d, represent constant symbols, and w, x, y, z, represent variables.

(a) {  $P(a, x, x)$ ,  
 $P(f(x,y), w, f(x, z)) \vee \neg P(y, w, z)$ ,  
 $\neg P(f(a, f(b, a)), f(c, a), x)$  }

(b) {  $Q(x, a, a)$ ,  
 $Q(x, f(x, y), z) \vee \neg Q(x, y, z)$ ,  
 $Q(x, f(y, z), f(y, w)) \vee \neg Q(x, z, w)$ ,  
 $\neg Q(b, f(b, f(c, f(b, a))), x)$  }

5. Let

$$\Phi_5 := \{ (\forall x)(\forall y)(\neg P(x,y) \vee R(x,f(x)) \vee R(f(x),x)),$$

$$(\forall x)(\forall y)(P(x,g(y)) \vee \neg Q(g(x),y)), (\forall x)(\forall y)(P(x,a) \vee Q(g(x),y)) \}$$

$$\varphi_5 := (\exists x)(\exists y)R(f(x),f(y))$$

Prove that  $\Phi_5 \models \varphi_5$  by converting this problem to an equivalent refutation problem, and then showing that the resulting set of clauses is unsatisfiable using resolution. Express your solution in the form of a proof graph, and include the substitutions which were used in the unifications.

6. Do exercise 9.10 on pages 207-208 of the textbook.
7. Do exercise 9.12 on page 209 of the textbook.
8. Do exercise 4 on pages 225-226 of the textbook.

If there is not time to do all of these exercises, some may be deferred until the final group exercise session on May 26, 1998.