

**Exercises for the group exercise session on
April 28, 1998**

Note: There may not be time to do all of these exercises in the time allotted. If there is one which gives you particular difficulty, come prepared to ask about it.

1. Consider the logical operation of Sheffer stroke (NAND) as described in the slides on propositional logic and in the textbook (page 13). Give left and right Gentzen-style proof rules for this operation. The rules should be both non-weakening and simplifying. In defining simplifying in this case, take the definition of the complexity of Sheffer stroke to be:

$$\text{Complexity}(\varphi_1 \mid \varphi_2) = 1 + \text{Complexity}(\varphi_1) + \text{Complexity}(\varphi_2)$$

2. Give proofs, in G' , of each of the following

(a) $\{A, \neg((\neg B \wedge A) \vee (\neg D \wedge C) \vee (\neg C \wedge B))\} \vDash D$.

(b) $\{(A \leftrightarrow B)\} \vDash \neg((\neg A \vee \neg B) \wedge (A \vee B))$.

(c) $\vDash (A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$

3. Using G' , determine whether or not each of the following formulas is satisfiable.

(a) $((A \wedge B \wedge C) \wedge (A \rightarrow \neg(B \vee C)) \wedge (D \leftrightarrow A) \wedge D)$

(b) $((P \wedge \neg P) \rightarrow (Q \wedge \neg Q))$

(c) $(A \wedge (A \rightarrow (B \vee C)) \wedge \neg(C \wedge \neg D \wedge \neg B))$

4. Repeat problems 2 and 3, this time using resolution.