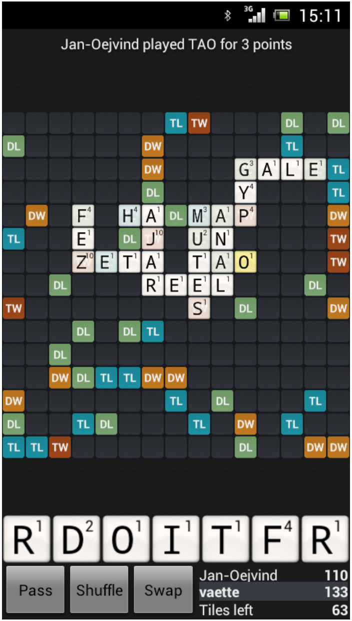
Scrabble!

- *The* classic word game, called Alfapet in Swedish.
- Invented by Alfred Mosher Butts in 1938.

Simple enough



A simple enough game:

- 2–4 players, each with 7 letter tiles.
- Players take turns placing 1–7 tiles on the 15x15 board s.t.:
  - The tiles placed form a horizontal and/or vertical line.
  - The line is “dense”, no gaps.
  - All words (gap-free lines) formed on the board are real English words.
- Score the words formed (points summed on all words created, modified by bonus squares).
- Turn ends by drawing new tiles.

What is the problem?

1. What is my highest-scoring next move?
2. What is my *best* next move ignoring opponents?
3. What are my best next k moves, given n tiles ignoring opponents?
4. What is my best move considering the opponent?

This gets very difficult. What can we reasonably do?

- Problem 1 is simplest. This ignores the opponent, the future state of the board and the tiles left.
    - Hopefully these can be reintroduced to a successful solution?
- Bonus tiles and letter scores are also complicated, we can try *generalising* these away:

$$f_{\text{score}}(\mathit{Board}, \mathit{Word}, \mathit{Placement}) \to \mathbb{N}$$

    - Assume that f_{score} is *efficient* (it is!)
    - But "forget" how it works to simplify the definition.
- Ignoring the future of the board makes its 2D-ness ignorable:

		H	A
B	E	E	
	N		
	D		

$\Longrightarrow$

*	*	H	A
	B	*	*
B	E	E	*
*	E	N	D

*	N	*	
H	E	*	*
	D		
A	*		

		H	A
B	E	E	
	N		
	D		

What kind of *-constraints are there?

c_1	c_2	H	A

- Single letters and empty positions are trivial constraints.
- c_1 says "Any letter that forms a word when followed by the letter *b*" (no such letter exists in English).
- c_2 says "Any letter which forms a word when followed by the word *end*" (b, f, l, m, p, r, s, or t).
- Also, constraints saying "Any letter α such that w and w' form a word when concatenated as $w\alpha w'$." are possible
  - w and w' are either (non-word) letters or words
  - For example, if *be* and *ate* are placed with a blank in between them the letter *r* may be placed to form the word *berate*.

The problem has three parts: (B, T, D) where

- B, the board, is a set of constraint sequences
  - 30 sequences of length 15 in classic scrabble
- T is a multi-set of letters from $\{a, \ldots, z\}$
  - 7 letters in classic scrabble
- D is a set of allowed words
  - More than 600,000 words in the Oxford English dictionary

So, $|D| \gg |B| + |T|$. A problem? Not really, *D is constant*.

Moves change B and T continuously, whereas the OED remains the same for decades at a time $\Longrightarrow$ we can do all the preprocessing we want on D (good!) but need to be careful with the complexity in D in practice (bad!)

Let us properly define the possible constraints. Let Σ be the alphabet and D the dictionary ($D \subset \Sigma^*$) in the following.

Definition (Prefix constraint)

If $w \in D \cup \Sigma$ and $\alpha \in \Sigma$ let $\alpha \in c_{prefix}(w, D)$ iff $\alpha w \in D$.

Definition (Suffix constraint)

If $w \in D \cup \Sigma$ and $\alpha \in \Sigma$ let $\alpha \in c_{suffix}(w, D)$ iff $w\alpha \in D$.

Definition (Infix constraint)

If $w \in D \cup \Sigma$ and $\alpha \in \Sigma$ let $\alpha \in c_{infix}(w, w', D)$ iff $w\alpha w' \in D$.

Not so interesting: $c_{blank} = \Sigma$, $c_{letter(\alpha)} = \alpha$.

Precomputation!

Since D seldom changes we can just precompute the contraints!

- Construct a hash-map $c_{prefix,D}$ which for all $w \in D$ gives $c_{prefix,D}(w) = c_{prefix}(D, w)$.
- Same for suffix.
- Almost the same for infix. Construct

$$c_{infix,D}(w, w') = c_{infix}(D, w, w') \text{ iff } c_{infix}(D, w, w') \neq \emptyset$$

  Leave other (w, w') unmapped.
  - All pairs would give a map of size $|D|^2$.
  - Statistical experiments suggest that merely tens of thousands of "valid" pairs exist in English.
  - Swedish and German might be worse?

With dictionary precomputations done in advance taking the board

		H	A
B	E	E	
	N		
	D		

it is not hard to construct an efficient algorithm to translate each row and column into a sequence of letters and sets of letters:

$$\begin{array}{ll}
(\emptyset, \{b, f, l, m, r, s, t, v, w\}, h, a), & (\Sigma, b, \{a, i, o\}, \{a, i\}), \\
(b, e, e, \{d, h, m, n, s, t, x, y\}), & (\{a\}, e, n, d)), \\
(\{e, y\}, n, \{m, n, p, r, s, w, x, y\}, \Sigma), & (h, e, \{o, u\}, \{o\}), \\
(\Sigma, d, \Sigma, \Sigma), & (a, \{f, n, p, r, s, t\}, \Sigma, \Sigma)
\end{array}$$

Finally, formalisation

With a fixed dictionary D, alphabet Σ, and scoring function f_{score}:

Definition (Scrabble problem instance)

A Scrabble problem instance is a tuple (B, T) where T is a finite multi-set over Σ and B is a finite subset of $(\Sigma \cup \mathcal{P}(\Sigma))^*$.

Definition (Placement)

A valid placement for a Scrabble problem instance (B, T) is a tuple $(\alpha_1 \cdots \alpha_n, \beta_1 \cdots \beta_m, i) \in D \times B \times \mathbb{N}$ such that

- $n + i < m$, and
- β_{i-1} and β_{n+i+1} are both sets if they exist, and
- $\alpha_j = \beta_{i+j}$ *or* $\alpha_j \in \beta_{i+j}$ for all $j \in \{1, \ldots, n\}$, and
- letting $P = \{\alpha_j \mid j \in \{1, \ldots, n\}, \beta_{i+j} \text{ is a set}\}$, $P \subseteq T$ and $P \neq \emptyset$.

Solutions and algorithms?

Definition (Scrabble problem solution)

For a Scrabble problem instance (B, T) a placement (w, b, i) is a solution if it maximises f_{score}.

We need to enumerate all possible placements (otherwise we need to analyse f_{score}). There shouldn't be *that* many (?)

1. Set $v_{\max} = 0$.
2. Take the next $b = \beta_1 \cdots \beta_m \in B$.
3. Intersect each set β_j with T.
4. If some set in b is empty, go to 2.
5. For each word w in D which matches b: $\ll$DANGER$\gg$.
    - Validate that (w, b, i) is a placement for some i.
    - Score (w, b, i) using f_{score}, set v to the score.
    - If $v > v_{\max}$ let $v_{\max} = v$.
6. Go to 2.

- Steps 1–4 and 6 are small stuff, $\mathcal{O}(|T| \sum\{|b| \mid b \in B\})$.
- A naive implementation of step 5 is trouble. $|B||D|$ attempts to match words to constraints.
- For now, assume few words match. Quick way to find them?

My first approach: hash-map indexing letter-pairs.

Dictionary lookup with letter pair indexing

Looking at this subproblem:

Definition

Given $b \in (\Sigma \cup \mathcal{P}(\Sigma))^*$ as input does there exist a word $w \in D$ such that w matches b in the sense of a placement?

Complexity in terms of $|D|$ still considered, but preprocessing of D is allowed.

Construct a hash-map $h : \Sigma \times \mathbb{N} \times \Sigma \to \mathcal{P}(D)$, which, for $\alpha_1, \alpha_2 \in \Sigma$ and $i \in \mathbb{N}$ gives $W = h(\alpha_1, i, \alpha_2)$ where W is all words which contain the letter α_1 and α_2 at positions i steps apart. That is,

$$h(x, 5, l) = \{\textit{anxiously}, \textit{expertly}, \textit{inexorably}, \textit{maximally}, \textit{obnoxiously}, \textit{paradoxically}, \textit{textually}, \textit{textural}\}$$

Pretty large but not difficult pre-processing.

The ispell american-english dictionary of 98,569 yields a map with 2,996,606 words. A key maps to an average of 492.54 words.

Dictionary lookup with letter pair indexing (cont'd)

Matching $(\Sigma, b, \{a, i, o\}, \{a, i\})$ can then be speeded up (?) by matching it to the words in

$$(h(b, 0, a) \cup h(b, 0, i) \cup h(b, 0, o)) \cap (h(b, 1, a) \cup h(b, 1, i))$$

instead of all of D (408 words in ispell american-english).

This depends on the structure of English words for speed. Hard to analyse. For other languages maybe h will map to gigantic classes?

Constructing the B-automaton

If we avoid relying on the structure of D then scanning may be necessary. How do we do scanning the best?

We can construct a deterministic finite automaton that attempts to match all of B at once!