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Modeling Immiscible Fluid Flow in Porous Media

This chapter provides an introduction to the models used in porous medium simulations. We begin with a definition of porous media, their basic properties and a motivation of macroscopic flow models. The subsequent sections are devoted to the development of models for single-phase flow and transport, multiphase flow and multiphase/multicomponent flows.

1.1 Porous Media

This subsection introduces the basic characteristics of porous media. Of special importance is the consideration of different length scales.

1.1.1 DEFINITIONS

A porous medium is a body composed of a persistent solid part, called *solid matrix*, and the remaining *void space* (or *pore space*) that can be filled with one or more fluids (e. g. water, oil and gas). Typical examples of a porous medium are soil, sand, cemented sandstone, karstic limestone, foam rubber, bread, lungs or kidneys.

A *phase* is defined in (Bear and Bachmat 1991) as a chemically homogeneous portion of a system under consideration that is separated from other such portions by a definite physical boundary. In the case of a *single-phase system* the void space of the porous medium is filled by a single fluid (e. g. water) or by several fluids completely *miscible* with each other (e. g. fresh water and salt water). In a *multiphase system* the void space is filled by two or more fluids that are *immiscible* with each other, i. e. they maintain a distinct boundary between them (e. g. water and oil). There may only be one gaseous phase since gases are always completely miscible. Formally the solid matrix of the porous medium can also be considered as a phase called the *solid phase*. Fig. 1.1 shows a two-dimensional cross section of a porous medium filled with water (single-phase system, left) or filled with water and oil (two-phase system, right).

Bear and Bachmat (1991) define a *component* to be part of a phase that is composed of an identifiable homogeneous chemical species or of an assembly of species (ions, molecules). The number of components needed to describe a phase is given by the *conceptual model*, i. e. it depends on the physical processes to be modeled. The example of fresh and salt water given above is described by a single-phase two component system.

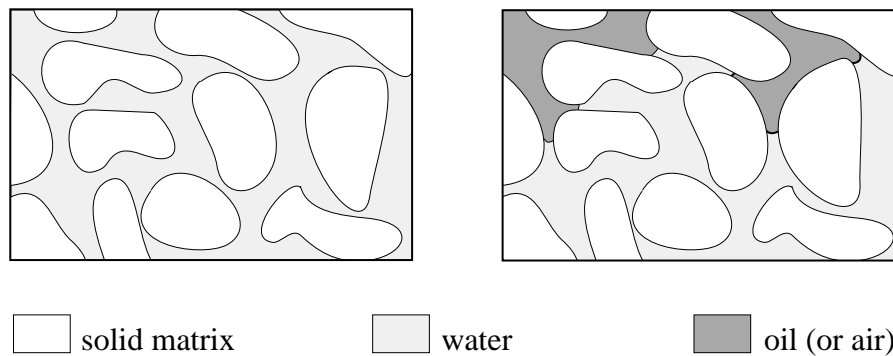


Figure 1.1: Schematic drawing of a porous medium filled with one or two fluids.

In order to derive mathematical models for fluid flow in porous media some restrictions are placed upon the geometry of the porous medium (Corey 1994, p. 1):

- (P1) The void space of the porous medium is interconnected.
- (P2) The dimensions of the void space must be large compared to the mean free path length¹ of the fluid molecules.
- (P3) The dimensions of the void space must be small enough so that the fluid flow is controlled by adhesive forces at fluid–solid interfaces and cohesive forces at fluid–fluid interfaces (multiphase systems).

The first assumption (P1) is obvious since no flow can take place in a disconnected void space. The second property (P2) will enable us to replace the fluid molecules in the void space by a hypothetical continuum (see next chapter). Finally, property (P3) excludes cases like a network of pipes from the definition of a porous medium.

1.1.2 CONTINUUM APPROACH

The important feature in modeling porous media flow is the consideration of different length scales. Fig. 1.2 shows a cross section through a porous medium consisting of different types of sands on three length scales.

In Fig. 1.2a the cross section is on the order of 10 meters wide. This scale is called the *macroscopic scale*. There we can identify several types of sand with different average grain sizes. A larger scale than the macroscopic scale is often called regional scale but is not considered here, see Helmig (1997).

If we zoom in to a scale of about 10^{-3} m as shown in Fig. 1.2b we arrive at the *microscopic scale* where individual sand grains and pore channels are visible.

¹The average distance a molecule travels between successive collisions with other molecules. Mean free path of air at standard temperature is about $6 \cdot 10^{-8}$ m.

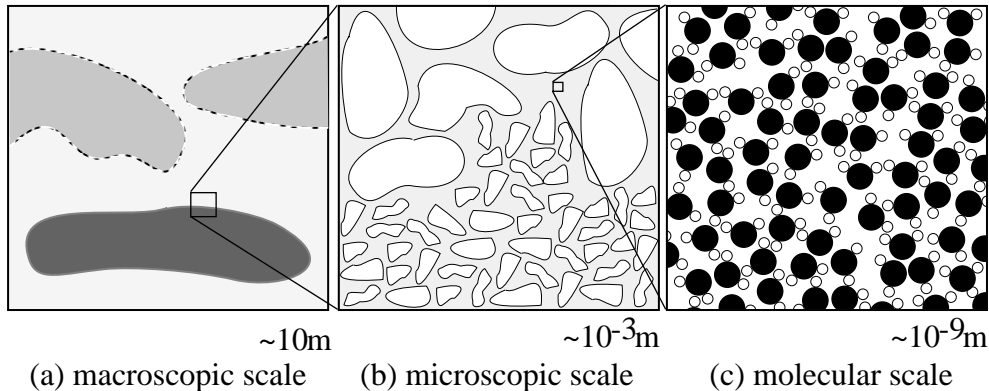


Figure 1.2: Different scales in a porous medium.

In the figure we see the transition zone from a fine sand to a coarser sand. The void space is supposed to be filled with water.

Magnifying further into the water-filled void space one would finally see individual water molecules as shown in Fig. 1.2c. The larger black circles are oxygen atoms, the smaller white circles are the hydrogen atoms. This scale of about 10^{-9}m will be referred to as the *molecular scale*.

It is important to note that the behavior of the flow is influenced by effects on all these different length scales. Fluid properties like viscosity, density, binary diffusion coefficient and miscibility are determined on the molecular scale by the individual properties of the molecules. On the microscopic scale the configuration of the void space influences the flow behavior through properties like the tortuosity of the flow channels or the pore size distribution, whereas on the macroscopic scale the large scale inhomogeneities play a rôle.

In classical continuum mechanics, see e. g. (Chung 1996), the individual molecules on the molecular scale are replaced by a hypothetical continuum on the microscopic scale. Quantities like mass (density) or velocity are now considered to be (piecewise) continuous functions in space and time. The continuum approach is a valid approximation if the mean free path length of the fluid molecules is much smaller than the physical domain of interest. This is ensured by property (P2) from the last subsection.

Accordingly, the flow of a single newtonian fluid in the void space of a porous medium is described on the microscopic level by the Navier–Stokes system of equations (cf. (Chung 1996)) with appropriate boundary conditions. However, the void space configuration is usually not known in such detail to make this description feasible. Moreover, a numerical simulation on that level is beyond the capabilities of today's computers and methods.

In order to derive a mathematical model on the macroscopic level another continuum is considered. Each point in the continuum on the macroscopic level is assigned average values over *elementary volumes* of quantities on the micro-

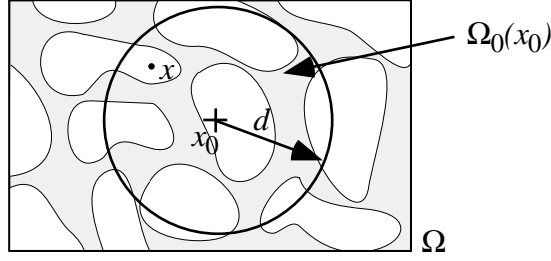


Figure 1.3: Illustration of the averaging volume.

scopic level. This process leads to *macroscopic equations* that do not need an exact description of the microscopic configuration. Only measurable statistical properties of the porous medium and the fluids are required.

1.1.3 REPRESENTATIVE ELEMENTARY VOLUME

The averaging process used for passing from the microscopic to the macroscopic level is illustrated for the *porosity*, a simple geometric property of the porous medium.

The porous medium is supposed to fill the domain Ω with volume $\text{meas}(\Omega)$. Let $\Omega_0(\mathbf{x}_0) \subset \Omega$ be a subdomain of Ω centered at the point \mathbf{x}_0 on the macroscopic level as shown in Fig. 1.3.

Further we define the void space indicator function on the microscopic level:

$$\gamma(\mathbf{x}) = \begin{cases} 1 & \mathbf{x} \in \text{void space} \\ 0 & \mathbf{x} \in \text{solid matrix} \end{cases} \quad \forall \mathbf{x} \in \Omega. \quad (1.1)$$

Then the porosity $\Phi(\mathbf{x}_0)$ at position \mathbf{x}_0 with respect to the averaging volume $\Omega_0(\mathbf{x}_0)$ is defined as

$$\Phi(\mathbf{x}_0) = \frac{1}{\text{meas}(\Omega_0(\mathbf{x}_0))} \int_{\Omega_0(\mathbf{x}_0)} \gamma(\mathbf{x}) d\mathbf{x} . \quad (1.2)$$

The macroscopic quantity porosity is obtained by averaging over the microscopic void space indicator function. If we plot the value of $\Phi(\mathbf{x}_0)$ at a fixed position \mathbf{x}_0 for different diameters d of the averaging volume $\Omega_0(\mathbf{x}_0)$ we observe a behavior as shown in Fig. 1.4. For very small averaging volumes the discontinuity of γ produces large variations in Φ , then at diameter l the average stabilizes and for averaging volumes with diameter larger than L the large scale inhomogeneities of the porous medium destabilize the average again, cf. (Bear and Bachmat 1991; Helmig 1997).

The averaging volume $\Omega_0(\mathbf{x}_0)$ is called a *representative elementary volume (REV)* if such length scales l and L as in Fig. 1.4 can be identified where the

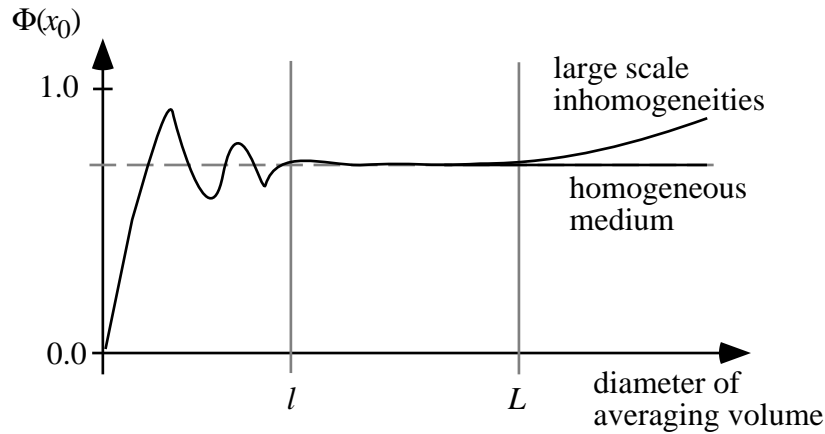


Figure 1.4: Porosity Φ for different sizes of averaging volumes.

value of the averaged quantity does not depend on the size of the averaging volume. In that case we can choose the averaging volume anywhere in the range

$$l \ll \text{diam}(\Omega_0(\mathbf{x}_0)) \ll L \quad . \quad (1.3)$$

If a REV cannot be identified for the porous medium at hand the macroscopic theories of fluid flow in porous media cannot be applied (Hassanizadeh and Gray 1979a).

The following table with typical values of porosity is taken from (Corey 1994):

Consolidated sandstones	0.1–0.3
Uniform spheres with minimal porosity packing	0.26
Uniform spheres with normal packing	0.35
Unconsolidated sands with normal packing	0.39–0.41
Soils with structure	0.45–0.55

1.1.4 HETEROGENEITY AND ANISOTROPY

A porous medium is said to be *homogeneous* with respect to a macroscopic (averaged) quantity if that parameter has the same value throughout the domain. Otherwise it is called *heterogeneous*. For example the porous medium shown in Fig. 1.5a has a different porosity in the parts with large and small sand grains and is therefore heterogeneous with respect to porosity.

Macroscopic tensorial quantities can also vary with direction, in that case the porous medium is called *anisotropic* with respect to that quantity. Otherwise it is called *isotropic*. As an example consider Fig. 1.5b. It is obvious that the porous medium is more resistive to fluid flow in the y -direction than in the x -direction. The corresponding macroscopic quantity called *permeability* will be anisotropic. Note that a similar effect as in Fig. 1.5b can also be achieved with the grain distribution shown in Fig. 1.5c.

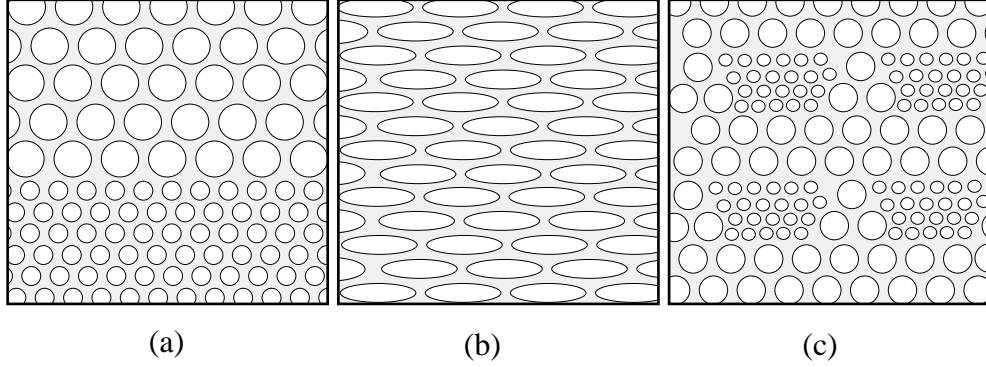


Figure 1.5: Porous media illustrating the concepts of heterogeneity and anisotropy.

1.2 Single-Phase Fluid Flow and Transport

In this subsection we consider macroscopic equations for flow and transport in porous media when the void space is filled with a single fluid, e. g. water, or several completely miscible fluids.

1.2.1 FLUID MASS CONSERVATION

Suppose that the porous medium fills the domain $\Omega \subset \mathbb{R}^3$, then the macroscopic fluid mass conservation is expressed by the partial differential equation

$$\frac{\partial(\Phi\rho)}{\partial t} + \nabla \cdot \{\rho\mathbf{u}\} = \rho q \quad \text{in } \Omega. \quad (1.4)$$

In its integral form this equation states that the rate of change of fluid mass in an arbitrary control volume $V \subseteq \Omega$ is equal to the net flow over the surface ∂V and the contribution of sources or sinks within V . The quantities in Eq. (1.4) have the following meaning.

- $\Phi(\mathbf{x})$ Porosity of the porous medium as defined in Eq. (1.2). It is a function of position in the case of heterogeneous media. In general it could depend on the fluid pressure (introduced below) or on time (e. g. swelling of clay) but these effects are not considered here.
- $\rho(\mathbf{x}, t)$ Density of the fluid given in $[kg/m^3]$. In this work density is either constant when the fluid is *incompressible* or we assume an equation of state for ideal gases where density is connected to fluid pressure p (see below):

$$p = \rho RT. \quad (1.5)$$

Here R is the individual gas constant and T the temperature in $[K]$, cf. (Helmig 1997). Note that the time derivative in Eq. 1.4 vanishes when the density is constant.

- $\mathbf{u}(\mathbf{x}, t)$ *Macroscopic apparent velocity* in $[m/s]$. This velocity is obtained by a macroscopic observer. On the microscopic level the flow takes only place through the pore channels of the porous medium where an average velocity of \mathbf{u}/Φ is observed.
- $q(x, t)$ Specific source/sink term with dimensions $[s^{-1}]$.

1.2.2 DARCY'S LAW

By using local averaging techniques, see e. g. (Whitaker 1986a), or homogenization, see (Hornung 1997), it can be shown that under appropriate assumptions (see below) the momentum conservation of the Navier–Stokes equation reduces to the Darcy–Law on the macroscopic level which is given by

$$\mathbf{u} = -\frac{\mathbf{K}}{\mu} (\nabla p - \rho \mathbf{g}). \quad (1.6)$$

This relation was discovered experimentally for the one-dimensional case by H. Darcy in 1856. It is basically a consequence of property (P3) of the porous medium. The new quantities in Eq. (1.6) have the following meaning.

- $p(x, t)$ Fluid *pressure* in $[Pa] = [N/m^2]$. This will be the unknown function to be determined by the flow model.
- \mathbf{g} Gravity vector pointing in the direction of gravity with size g (gravitational acceleration). Dimension is $[m/s^2]$. When the z -coordinate points upward we have $\mathbf{g} = (0, 0, -9.81)^T$.
- $\mathbf{K}(\mathbf{x})$ Symmetric tensor of *absolute permeability* with dimensions $[m^2]$. It is a parameter of the solid matrix only and may depend on position in the case of a heterogeneous porous medium. Furthermore \mathbf{K} may be anisotropic if the porous medium has a preferred flow direction as explained in subsection 1.1.4.
- $\mu(\mathbf{x}, t)$ *Dynamic viscosity* of the fluid given in $[Pa \cdot s]$. In the applications considered here μ is either constant or a function of pressure.

Darcy's Law is valid for the slow flow (inertial effects can be neglected) of a Newtonian fluid through a porous medium with rigid solid matrix. No slip boundary conditions are assumed at the fluid–solid boundary on the microscopic level. For details we refer to (Bear 1972; Whitaker 1986a; Whitaker 1986b; Hassanizadeh and Gray 1979a; Hassanizadeh and Gray 1979b; Hassanizadeh and Gray 1980).

Inserting Eq. (1.6) into Eq. (1.4) yields a single equation for the fluid pressure p ,

$$\frac{\partial(\Phi p)}{\partial t} - \nabla \cdot \left\{ \rho \frac{\mathbf{K}}{\mu} (\nabla p - \rho \mathbf{g}) \right\} = \rho q \quad \text{in } \Omega \quad (1.7)$$

with initial and boundary conditions

$$p(\mathbf{x}, 0) = p_0(\mathbf{x}) \quad \text{in } \Omega, \quad (1.8a)$$

$$p(\mathbf{x}, t) = p_d(\mathbf{x}, t) \quad \text{on } \Gamma_d, \quad \rho \mathbf{u} \cdot \mathbf{n} = \phi(\mathbf{x}, t) \quad \text{on } \Gamma_n. \quad (1.8b)$$

In the case of a compressible fluid Eq. (1.7) is of parabolic type, in the incompressible case it is of elliptic type (then the initial condition (1.8a) is not necessary).