Institutionen för datavetenskap
Umeå universitet

December 8, 2010
Numeriska metoder för civilingenjörer

Theme: Interpolation and Quadrature
Part II

Rules:

- Each student should hand in individually completed solution.
- Use the provided answer sheet and add printouts of pictures and source code as requested.
- You may discuss the problem among fellow students. If you receive considerable help from someone, say so in your solutions.
- Do not copy solutions or code from others. Do not lend your solution or your code to other students.
- A correct solution submitted at the latest **December 20 at 12.00 (noon)** is worth 2 bonus point on the final exam. An almost correct solution submitted in time will also entitle to bonus points, provided the solution is corrected before the final exam.

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1. Implement the trapezoidal and the Simpson rule, and numerically evaluate the integrals

$$I_1 = \int_0^1 \mathrm{e}^x \,\mathrm{d}x, \text{ and } I_2 = \int_0^1 \sqrt{x}\,\mathrm{d}x$$

   using step lengths $h = 2^{-k}$ for $k = 1, 2, \ldots, 6$. Estimate the order of accuracy; that is, find the constant p so that the error, as the step length is varied, behaves as ch^p, where c is a constant.

2. Use MATLAB's build in adaptive quadrature function `quad` to evaluate the integrals above. How many function evaluations are required to evaluate I_1 and I_2, respectively, when using the error tolerance 10^{-10} ? Use the syntax `[int,fcnt] = quad(f,a,b,tol)` to obtain the number of function evaluations `fcnt`. See `help quad` for more information.

3. Interpolate the function

$$f(x) = |x|$$

   in the interval $-1 \le x \le 1$ using

   (a) polynomial interpolation with N equispaced interpolation points,

   (b) polynomial interpolation with N Chebyshev interpolation points.

   Use Matlab's functions `polyfit` and `polyval` with $N = 5$, 10, 20, and 40 points.

4. The spline interpolation procedure discussed in class produces curves of the form $(x, s(x))$. However, many curves (closed curves, for instance) cannot be written in this form but require the general parametric form $(s_1(t), s_2(t))$. A general point set (x_i, y_i) $i = 0, \ldots, n$ thus needs to be interpolated with *two* separate splines s_1, s_2, one for each coordinate.

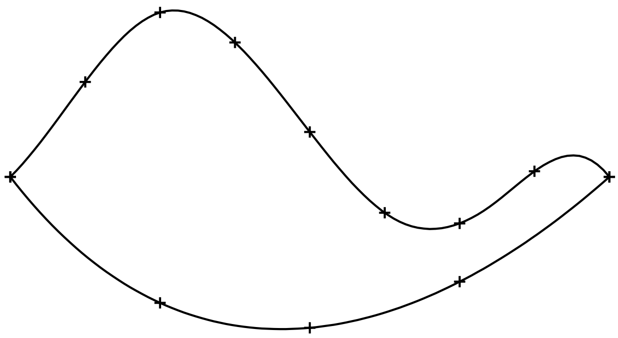

Figure 1: Plan sketch of object for which we are interested in finding the center of mass.

Now assume that we sample the spiraling function

$$f(t) = \begin{pmatrix} t\cos(t) \\ t\sin(t) \end{pmatrix}$$

at the points $f(i/2)$ for $i = 0,\ldots,20$. Use the Matlab command `spline` to construct the cubic spline interpolants s_1 and s_2 that interpolate the above 21 points. Plot the interpolant and the points in the same figure.

5. Compute the center of mass of the object in Figure 1 using cubic splines for the interpolation of f_1 and f_2. The data positions of the points marked with plus signs in the sketch can be downloaded from the course homepage[1] and is summarized in the table below.

x	1.000	1.125	1.250	1.375	1.500	1.625	1.750	1.875	2.000
$f_1(x)$	1.000		0.789		0.748		0.823		1.000
$f_2(x)$	1.000	1.159	1.275	1.225	1.075	0.940	0.922	1.009	1.000

Hint: For each integral $\int_a^b g(x)\,\mathrm{d}x$ that you need to compute, write a Matlab function that given a *vector* of x values returns corresponding *vector* of values $g(x)$. Use the `spline` function to construct/evaluate each function g from the sampled values above. Compute the integrals using the `quad` function.

[1] The files `f1.dat` and `f2.dat` are located in the directory `http://www.cs.umu.se/kurser/5DV040/HT10/themes.php` and contain the coordinates for the functions f_1 and f_2, respectively.