Institutionen för datavetenskap
Umeå universitet

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Numeriska metoder för civilingenjörer

Theme: Soft soils and nonlinear equations
Part I

Study the theory (the lecture notes and relevant sections in the book), Part I below, and complete the preparatory exercises *before* the start of the lab. The preparatory exercises need to be completed to get a passing grade on the lab, so show them to the teacher in order to verify that they are completed. At the beginning of the lab, Part II of the theme, the computer exercises, will be handed out and uploaded to the home page of the course. General rules for the preparatory exercises and the computer exercises:

- Each student should hand in individually completed solutions.
- You may discuss the problem among fellow students. If you receive considerable help from someone, say so in your solutions.
- Do not copy solutions or code from others. Do not lend your solution or code to other students.

Soft soil stiffness model[1]

Assume that a layer of soft homogeneous soil of depth D lies on top of a base of harder soil or rock. We wish to determine the pressure needed to sink a large, stiff, and circular plate of radius r a distance $d < D$ into the soil. A simplified model for the required pressure p is

$$p = a_1 e^{a_2 r} + a_3 r, \tag{1}$$

where a_1, a_2, and a_3 are constants with $a_2 > 0$. The constants depend on the material properties of the soil, the depth d, but not on the radius r of the object. To determine parameters a_1, a_2, and a_3, three small plates with different radii are sunk to the same depth, and the pressures required for this sinkage are measured. Substituting the measurements data into expression (1) leads to a system of nonlinear equations for parameters a_1, a_2, and a_3. Once the parameters have been determined, formula (1) can then be used, for instance, to estimate the minimal size of a plate required to sustain a large load.

Remark. In this lab, we use the bare minimum of only three measurements to determine parameters a_1, a_2, and a_3. A more robust strategy, to be recommended in a real-world scenario, is to use more plates of various radii and determine the coefficients in a (nonlinear) least-squares sense.

[1] The soil problem is based on problem 7 in exercise set 10.2 of R. L. Burden and J. D. Faires, *Numerical Analysis, Fourth Edition*, PWS-KENT Publishing Company, 1989. Burden and Faires refer to M. G. Bekker, *Introduction to terrain vehicle systems*, University of Michigan Press, 1969, pp. 89–94, for background information on the modeling.

Preparatory exercises for lab

1. Write down the nonlinear system of equations that emerges from the above strategy using three measurements to determine parameters a_1, a_2, and a_3 in equation (1). Also compute the Jacobian matrix.

2. Let a sequence of real numbers be defined by

$$x_{k+1} = x_k(2 - dx_k),$$

where $d \neq 0$ is a given number. Assume that the sequence converges to a nonzero number.

   (a) To what number does the sequence converge?

   (b) Show that the convergence rate is quadratic.