5DV037 Fundamentals of Computer Science Fall 2010
Obligatory Exercise 5
Due date: October 25, 2010 at 8am (0800)

In the problems below, recall that DTM_Σ denotes the encodings of all DTMs (deterministic Turing machines) over the alphabet Σ, and f_M denotes the (possibly partial) function of one variable computed by the DTM M.

1. Let $\Sigma = \{0, 1\}$, and define $L_1 = \{M \in \mathsf{DTM}_\Sigma \mid f_M(i) \text{ is defined for at least one } i \in \mathbb{N}\}$. Assume that $i \in \mathbb{N}$ is represented in binary, as a string in $\{0, 1\}^*$.

   (a) State whether or not this language is decidable, and justify your answer with arguments based upon ideas developed in the textbook and/or lecture slides.

   (b) State whether or not this language is semidecidable (Turing acceptable), and justify your answer with arguments based upon ideas developed in the textbook and/or lecture slides.

2. Let $\Sigma = \{0, 1\}$, and define $L_2 = \{M \in \mathsf{DTM}_\Sigma \mid f_M(i) \text{ is defined for at most one } i \in \mathbb{N}\}$. Assume that $i \in \mathbb{N}$ is represented in binary, as a string in $\{0, 1\}^*$.

   Repeat the two parts of question 1 for this language.

   Hint for part (b): Consider the complement of the property defined by the language.

3. In the context of DTMs over an alphabet Σ, consider the following question: Given an arbitrary DTM $M = (Q, \Sigma, \Gamma, \delta, q_0, \square, F)$ and an arbitrary $\alpha \in \Sigma$, starting from the configuration $\mathcal{I}\langle M, \alpha\rangle$ (that is, from the initial state q_0 with α on the tape), does M ever reach a configuration containing one of its accepting states during the computation? State whether or not this property is decidable, and justify your answer with arguments based upon ideas developed in the textbook and/or lecture slides.

Further Notes:

1. As stipulated in the course syllabus, this exercise may be done either individually, in a group of two, or in a group of three. Remember that there are point penalties for late submission. See the course syllabus.
2. It is strongly recommended that you use a graphical tool to display your results. If you draw them by hand, they must be very neat. It is not allowed to copy the work of others. The submission must be the original work of the individual(s) in the working group. The grader reserves the right to interview members of the working group about the solution.
3. So that solutions may be discussed in a class meeting, students and/or groups that are very late in preparing a solution may be required to solve an alternate problem to receive credit for this exercise.

Updated October 21, 2010