5DV037 Fundamentals of Computer Science Fall 2010
Obligatory Exercise 2
Due date: September 27, 2010 at 8am (0800)

1. Let $r = ((a+b)^* + (a \cdot (b+c))^*)^*$. Using the algorithm given in the lecture slides or in the textbook, construct an NFA which accepts the language defined by r. Your solution must follow the algorithm.

2. Give a right-linear grammar which generates precisely the hex representations for integers in the standard format ±`0xnn...nn` with each `n` is a hex "digit" in $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, a, b, c, d, e, f, A, B, C, D, E, F\}$. The number of such symbols after the `x` is not bounded but there must be at least one. The sign may be `+`, `-`, or missing. Examples include `0x4A7bc`, `+0x1234`, and `-0xf`.

3. Let

$$L = \{\alpha \in \{a, b\}^* \mid |\mathsf{Count}\langle a, \alpha\rangle - \mathsf{Count}\langle b, \alpha\rangle| \leq 10\}$$

That is, L is the set of all strings over $\{a, b\}$ for which the number of a's differs from the number of b's by at most ten. Show that L is not a regular language.

4. The programming language APL has no implicit operator hierarchy. Rather, expressions are evaluated from right to left. Thus, the expression `3*2+1` evaluates to `9` and not `7`. Parentheses may be used for grouping, so `(3*2)+1` evaluates to `7`. Give an unambiguous context-free grammar for the expressions involving the binary operations `+` and `*`, including those with parentheses, which respects right-to-left evaluation. Thus, the derivation tree for the expression `3*2+1` should represent `3` in its left subtree and `2+1` in its right subtree, with `*` the operator for combining them. On the other hand, the derivation tree for the expression `(3*2)+1` should represent `(3*2)` in its left subtree and `1` in its right subtree, with `+` the operator for combining them.
For simplicity, assume that the basic elements to which operations are applied are the single-digit integers; *i.e.*, $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
In addition to the grammar itself, show the derivation trees for each of the three expressions `(3*2)+1`, `3*(2+1)`, and `3*2+1`.
(Hint: Look at the grammars given in the lecture slides on CFLs and CFGs for arithmetic expressions and if-then-else statements and adapt the representations shown there.)

Further Notes:

1. As stipulated in the course syllabus, this exercise may be done either individually, in a group of two, or in a group of three. Remember that there are point penalties for late submission. See the course syllabus.
2. It is strongly recommended that you use a graphical tool to display your results. If you draw them by hand, they must be very neat. It is not allowed to copy the work of others. The submission must be the original work of the individual(s) in the working group. The grader reserves the right to interview members of the working group about the solution.

3. So that solutions may be discussed in a class meeting, students and/or groups that are very late in preparing a solution may be required to solve an alternate problem to receive credit for this exercise.