Umeå University
Department of Computing Science
5DV037 — Fundamentals of Computer Science
Examination: January 4, 2011

Name (printed): ______________________________

Swedish ID number: ______________________________

Computer User-ID: ______________________________

Signature: ______________________________

Secret code number: ______________________________

Instructions: / Instruktioner:

This examination will be graded anonymously. This page will be removed before the instructor receives the examination for grading. The secret code number given above must therefore be written on every answer page which you turn in to the examination proctor.

Denna skrivning rättas kodad. Detta blad kommer att avskiljas innan läraren får skrivningen för rättning. Ovanstående kod måste därför finnas på samtliga svarsblad när du lämnar skrivningen till skrivvakten.

To the proctor of the examination: / Till skrivningsbevakaren:

Detach this cover sheet from the examination and put it in the envelope which is addressed to Yvonne Löwstedt, Department of Computing Science.

Avskilj detta försättsblad och stoppa i kuvert som skickas till Yvonne Löwstedt, Datavetenskap.

Umeå University
Department of Computing Science
5DV037 — Fundamentals of Computer Science
Examination: January 4, 2011

Secret code number: ______________________________

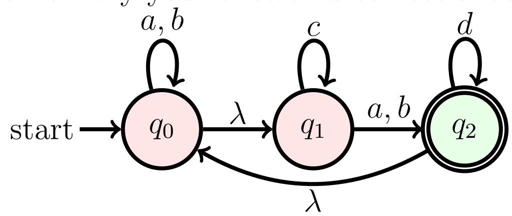
1. Answers may be written in English or Swedish. However, all technical terms which do not have an absolutely standard representation in Swedish must be given in English.
2. An English/X - X/English dictionary may be used. No other help materials are allowed.
3. Answers must be written on the official university answer sheets which are provided. Collate the answer sheets in numerical order of the problems, and write on only one side of the paper. Write only the question number and your secret code number on these pages; do not write your name or ID.
4. Show your work. For questions which require that an answer be computed, answers without derivations will not receive full credit.
5. The examination has a total of 1000 points.
6. For problems with multiple parts, you have the choice, for each part, to do the problem, or to skip it for partial credit. In the table below, place an X in the position for any problem for which you have attempted a solution, and which you wish to have graded. It is extremely important that you fill in this table properly, because of the following option. For any box which is left blank, the associated question will not be graded, and you will instead be awarded 15% of the points for that question. Your decision to leave a box blank is definitive, so be very careful. For example, If you leave box 8(b) blank, your answer to that question will not be graded, even if it is completely correct. On the other hand, if you place an X in box 8(b), but provide no answer whatsoever to that question, you will not receive 15% of the points for that question. It is strongly recommended that you use a pencil, in case you change your mind!

Prob	1	2	3	4	5	6	7	8	9	10
(a)										
(b)	■	■	■			■	■			
(c)	■	■	■	■		■	■			
(d)	■	■	■	■	■	■	■			
(e)	■	■	■	■	■	■	■		■	

(1: 100 points) Let L be the language consisting of all strings in $\{a, b\}^*$ which end in ab. In other words, $L = \{\alpha \in \{a, b\}^* \mid \alpha = \beta ab \text{ for some } \beta \in \{a, b\}^*\}$. Design a DFA (deterministic finite-state automaton) which accepts L. Specify your result as a state-transition diagram and provide a brief explanation of why your answer is correct.

(2: 100 points) Let L be the language consisting of all strings in $\{a, b\}^*$ which end in $ababababab$. In other words, $L = \{\alpha \in \{a, b\}^* \mid \alpha = \beta ababababab \text{ for some } \beta \in \{a, b\}^*\}$. Design an NFA (nondeterministic finite-state automaton) which accepts L. Specify your result as a state-transition diagram and provide a brief explanation of why your answer is correct.

(3: 100 points) Give a regular expression which describes the language accepted by the NFA depicted below. Your answer need not be the result of following the algorithm given in the course, but a clear explanation of why your answer is correct should be provided.



(4: 100 points total)

(a: 50 points) State the pumping lemma for regular languages. The statement must be precise and unambiguous.

(b: 50 points) Using the pumping lemma for regular languages, show that $L = \{a^i ba^i \mid i \geq 0\}$ is not regular.

(5: 100 points total) Let $L = \{a^i ba^i \mid i \geq 0\}$.

(a: 50 points) Give a CFG (context-free grammar) which generates L.

(b: 25 points) For your grammar of (a), show a parse tree for the string $aabaa$.

(c: 25 points) If your grammar of (a) has at least two parse trees for $aabaa$, give a second parse tree. If it has only one parse tree, explain clearly why this is the case.

(6: 100 points total) State the pumping lemma for context-free languages. The statement must be reasonably formal and precise.

(7: 100 points total) Let $G = (V, \Sigma, S, P)$ be the context-free grammar defined by

$$
\begin{aligned}
V &= \{S, A, B, C\} \\
\Sigma &= \{a, b, c\} \\
P = \{\ \ & S \to AC \mid BcB \\
& A \to aA \mid aBC \mid abc \\
& B \to bbB \mid b \\
& C \to AC \mid BC
\end{aligned}
$$

Using the algorithm given in the textbook or in the course slides, construct an equivalent reduced context-free grammar (*i.e.*, with no useless variables or productions). To obtain credit, you must show your work.

(8: 100 points total) In the following, it may be assumed that that the definition of a deterministic Turing machine (DTM) is known.

(a: 20 points) Explain what it means for a DTM $M = (Q, \Sigma, \Gamma, \delta, q_0, \sqcup, F)$ to accept a string $\alpha \in \Sigma^*$.

(b: 20 points) Explain the two ways in which a DTM $M = (Q, \Sigma, \Gamma, \delta, q_0, \sqcup, F)$ can reject a string $\alpha \in \Sigma^*$.

(c: 20 points) Explain what it means for a language L to be decidable.

(d: 20 points) Explain what it means for a language L to be semidecidable (or Turing acceptable).

(e: 20 points) Define the halting problem for DTMs.

(9: 100 points total) Let $\Sigma = \{a, b\}$. In each of the questions below, in addition to the requested language, provide a brief explanation of why it has the required property.

(a: 25 points) Give an example of a language L over Σ which is decidable.

(b: 25 points) Give an example of a language L over Σ which is semidecidable but not decidable.

(c: 25 points) Give an example of a language L over Σ which is is not semidecidable but whose complement $\{a, b\}^* \setminus L$ is semidecidable.

(d: 25 points) Give an example of a language L over Σ which is is not semidecidable and whose complement $\{a, b\}^* \setminus L$ is not semidecidable either.

(10: 100 points total) Provide brief descriptions of the following concepts.

(a: 20 points) The class $\mathcal{P}$.

(b: 20 points) The class $\mathcal{NP}$.

(c: 20 points) The notion of problem reduction.

(d: 20 points) The class of $\mathcal{NPC}$ of $\mathcal{NP}$-complete problems.

(e: 20 points) Cook's Theorem.