

Laboration 1: Elementary programming in MATLAB

1 Introduction

The purpose of this laboration is to practice some elementary MATLAB programming. This particular lab is compulsory also for students taking the course *Analys i flera variabler 4p* that runs parallel to this course, thus the choice of mathematical applications in the exercises.

Exercises 4 and 5 are the main exercises of the lab. The tasks are to write a MATLAB code that produces a graphical visualization of a given vector field (taken from physics) and to calculate a given path integral that involves the vector field. These exercises will be presented, by one group of students to another, at the seminar on the last day of the course.

2 Exercises

Exercise 1. Write a function `mean(X)` that receives a vector of numbers $X = [x_1, x_2, \dots, x_N]$ and returns the mean value $\frac{1}{N} \sum_{n=1}^N x_n$.

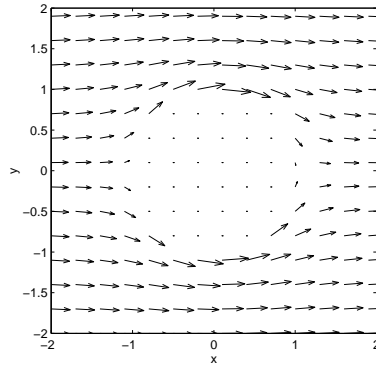
Exercise 2. Write a function `negate(X)` that receives a vector $X = [x_1, x_2, x_3, x_4, \dots, x_N]$ and returns the same vector, but with every second element negated, i.e. $X = [x_1, -x_2, x_3, -x_4, \dots, x_N]$.

Exercise 3. Produce a graphical visualization (using the command `surf`) of the scalar field $f(x, y) = e^{-(1-xy)^2}$ for $x \in [-3, 3]$ $y \in [-3, 3]$.

Exercise 4. Produce a graphical visualization (using the command `quiver`) of the vector field given below. The field should be calculated by looping over the grid and evaluating the field components at each grid point. Furthermore, the solutions should consist of *at least two MATLAB functions* (one being called on by the other) that you write yourselves. Depending on which group you are assigned to (an **A**-group or a **B**-group) the vector field is:

A The velocity field \mathbf{u} of fluid streaming past a cylindrical object. (The fluid velocity *far* from the cylinder is \mathbf{U} .)

$$\mathbf{u} = \begin{cases} \left(1 + \frac{1}{2} \frac{R^3}{r^3}\right) \mathbf{U} - \frac{3}{2} \frac{R^3}{r^3} \frac{\mathbf{U} \cdot \mathbf{r}}{r^2} \mathbf{r} & , r \geq R \\ 0 & , r < R \end{cases} \quad (1)$$



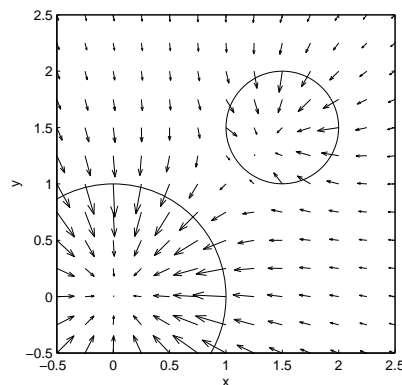
The fluid flow around a cylinder.

where $\mathbf{u} = (u_x, u_y)$, $\mathbf{r} = (x, y)$, $r = |\mathbf{r}|$ and $\mathbf{U} \cdot \mathbf{r}$ denotes the scalar product between \mathbf{U} and \mathbf{r} . Use $x \in [-2, 2]$, $y \in [-2, 2]$, $\mathbf{U} = (1, 0)$ and $R = 1$.

B The gravitational field from a planet and its moon is given by $\mathbf{g} = \mathbf{g}_1 + \mathbf{g}_2$,

$$\mathbf{g}_i(\mathbf{x}) = \begin{cases} -M_i G \frac{\mathbf{x} - \mathbf{x}_i}{|\mathbf{x} - \mathbf{x}_i|^3} & , |\mathbf{x} - \mathbf{x}_i| \geq R_i \\ -M_i G \frac{\mathbf{x} - \mathbf{x}_i}{R_i^3} & , |\mathbf{x} - \mathbf{x}_i| < R_i \end{cases} \quad (2)$$

where $\mathbf{x} = (x, y, z)$ is the position vector, M_i is the mass and R_i the radius of the body i ($i = 1$ or 2).



The gravitational field inside and around a planet and its moon when they have comparable masses and are close to each other.

The gravitational field from the planet is given by \mathbf{g}_1 ($i = 1$) and the moons gravitational field is \mathbf{g}_2 ($i = 2$). Use the values $\mathbf{x}_1 = (0, 0, 0)$, $R_1 = 1$, $M_1 = 1$, $\mathbf{x}_2 = (1.5, 1.5, 0)$, $R_2 = 0.5$, $M_2 = M_1 (R_2/R_1)^3$ and $G = 1$. Visualize the field in the xy -plane over the region $x \in [-0.5, 2.5]$, $y \in [-0.5, 2.5]$ (use $z = 0$).

Exercise 5. Calculate the path integral (**A** or **B**) given below numerically. A simple numerical method can be obtained in the following way. Consider an integral $I = \int_C \mathbf{F} \cdot d\mathbf{s}$ along a curve C , where $\mathbf{F} = (F_x, F_y)$ is a vector field. Say that we have already constructed a quadratic grid over a region containing C and that the matrices \mathbf{F}_x and \mathbf{F}_y corresponds to the field components in the grid points. If the grid is fine enough the integral is approximately given by

$$I \approx \sum_{(n,m)} \mathbf{F}_x(n,m) \Delta s_x + \mathbf{F}_y(n,m) \Delta s_y \quad (3)$$

where the summation should be taken over those grid points (n, m) that trace out the curve C . Δs_x and Δs_y stands for the step-length (grid point distance) in the x - and y -direction, respectively, as given by $d\mathbf{x} = (\Delta s_x, \Delta s_y)$.

A Use the fluid velocity \mathbf{u} given by Eq. (1). Calculate numerically

$$\int_C \mathbf{u} \cdot d\mathbf{s} \quad (4)$$

along the straight line: (x, y) from $(4, 4)$ to $(4, -4)$ with $d\mathbf{s} = (-dy, 0)$. Confirm that the same result is obtained (approximately) if one integrates along the line (x, y) which goes straight through the cylinder from $(0, 4)$ to $(0, -4)$, with $d\mathbf{s} = (-dy, 0)$ by numerical calculation¹. Are the results identical? If not, explain why and try to improve the result.

B Use the gravitational field \mathbf{g} given by Eq. (2). Calculate numerically

$$\int_C \mathbf{g} \cdot d\mathbf{s} \quad (5)$$

and confirm that the result is zero² for the curve C being the straight line (x, y) from $(-0.5, 2.5)$ to $(2.5, -0.5)$ with $d\mathbf{s} = (dx, dy)$.

¹The integral (4) gives the total fluid flow (per unit time) across the curve C . That is why $d\mathbf{s}$ is directed perpendicular to the curve, and not parallel to it. Apparently the total flow across the two curves is the same, although fluid flows across a *shorter* length along the latter curve. This is compensated by that the flow velocity is *larger* when passing the cylinder.

²The integral 5 corresponds to the amount of energy that is gained (or required) by moving an object along the path in the given gravitational field. The energy is zero for this particular path since the start and end points have the same potential energy (they are both located at the same distances from the planet and from the moon).

3 Hints

Sometimes your plot does not appear as desired. MATLAB makes its own choices of perspective, scaling of the axes, coloring etc. Here follows some ways of altering the appearance of the content in the graphics window. Note that some of these changes can also be done directly through the menus in the graphics window.

- **daspect**. Sometimes MATLAB stretches the figures, e.g. making a sphere appear like an ellipsoid. Type `daspect([1,1,1])` to obtain equal scaling on all three axis.
- **meshgrid**. A too fine grid can make the plots messy. A too sparse grid may lead to that some information is lost.
- **axis**. To change the ranges of the axes use the command `axis([XMIN XMAX YMIN YMAX ZMIN ZMAX])`.
- The commands **view**, **colormap** and **shading** are used to set the perspective, coloring and shading effects.
- The circles in the figures above are drawn with the command **rectangle** (it is not compulsory to included them in your plots).

4 Presentation

At the last day of the course a seminar will take place, where the members of each group will together present the laboration to some other groups and the lecturers. Which group and which Exercise (4 or 5) will be decided randomly at the beginning of the seminar session. The presentation

- should last for *precisely* 15 minutes.
- should be directed mainly towards the students of the other groups.
- should describe how you solved the exercise and what the result was.
- will be followed by a discussion where the listening group should ask questions and make comments.