5DV022 Problem Exercise 5 Fall 2008

Solutions to this assignment are due on October 13, 2008 at class time. The signed cover sheet must be turned in with your solutions.

Turn in your solutions to the course instructor or place them in ihs mailbox. Do not put them in the mailboxes for laboratory reports.

Given is the following matrix representation for a directed graph.

$$\begin{bmatrix} \infty & 7 & 3 & 12 & 8 \\ 3 & \infty & 6 & 14 & 9 \\ 5 & 8 & \infty & 6 & 18 \\ 9 & 3 & 5 & \infty & 11 \\ 18 & 14 & 9 & 8 & \infty \end{bmatrix}$$

Using the algorithm described in Section 6.4 of the course notes (or, equivalently, in Section 8.3 of the textbook), find an optimal solution for the travelling salesman problem defined by this graph. In your solution, list the vertices of the corresponding search tree in order of generation. For each such vertex, give the following.

(a) The index of its generation.

(b) The path in the graph to which the vertex corresponds.

(c) The value $\hat{c}$ of the node, expressed as a sum of the value of its parent, the cost of the edge corresponding to travelling from its parent, the amount added by row reduction, and the amount added by column reduction. (Rows must be reduced before columns.) In the case that it is a near-leaf, add in the cost of the final path back to the start vertex as well. (For the root node, there will not be any row and column reduction.)

(d) Four versions of the associated cost matrix.

- (i) before any reduction whatever;
- (ii) before any row or column reduction, but after the assignment of certain entries to the value of ∞ from dynamic reduction (not applicable to the root vertex);
- (iii) after row reduction but before column reduction, with the value by which each row was reduced;
- (iv) after both row and column reduction, with the value by which each column was reduced.

Only those vertices actually generated in the branch-and-bound search need be listed. To receive credit, your answers must be in the format described above. Shown on the next two pages is a partial solution, in this format, of the first six vertices generated of the example in the course notes and textbook.

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The LaTeX source for this solution is available on the course web page. If you know how to use LaTeX, it should be easy to plug your numbers into the template provided.

Vertex 1; path 1; $\hat{c} = 21 + 4 = 25$:

$$\begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 14 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix} \mapsto \begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix} \begin{array}{c} 10 \\ 2 \\ 2 \\ 3 \\ 4 \\ \hline 21 \end{array} \mapsto \begin{array}{c} \begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix} \\ \begin{matrix} 1 & 0 & 3 & 0 & 0 \end{matrix} \; - \; 4 \end{array}$$

Vertex 2; path 12; $\hat{c} = 25 + 10 + 0 + 0 = 35$:

$$\begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix} \mapsto \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix} \mapsto \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix} \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \end{array} \mapsto \begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix} \\ \begin{matrix} 0 & 0 & 0 & 0 & 0 \end{matrix} \; - \; 0 \end{array}$$

Vertex 3; path 13; $\hat{c} = 25 + 17 + 0 + 11 = 53$:

$$\begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix} \mapsto \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{bmatrix} \mapsto \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{bmatrix} \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \end{array} \mapsto \begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 4 & 3 & \infty & \infty & 0 \\ 0 & 0 & \infty & 12 & \infty \end{bmatrix} \\ \begin{matrix} 11 & 0 & 0 & 0 & 0 \end{matrix} \; - \; 11 \end{array}$$

Vertex 4; path 14; $\hat{c} = 25 + 0 + 0 + 0 = 25$:

$$\begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix} \mapsto \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix} \mapsto \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix} \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \end{array} \mapsto \begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix} \\ \begin{matrix} 0 & 0 & 0 & 0 & 0 \end{matrix} \; - \; 0 \end{array}$$

Vertex 5; path 15; $\hat{c} = 25 + 1 + 0 + 5 = 31$:

$$\begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix} \mapsto \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & 2 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 15 & 3 & 12 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix} \mapsto \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 10 & \infty & 9 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 9 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix} \begin{array}{c} 0 \\ 2 \\ 0 \\ 3 \\ 0 \\ \hline 5 \end{array} \mapsto \begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 10 & \infty & 9 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 9 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix} \\ \begin{matrix} 0 & 0 & 0 & 0 & 0 \end{matrix} \; - \; 0 \end{array}$$

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Vertex 6; path 142; $\hat{c} = 25 + 3 + 0 + 0 = 28$:

$$\begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix} \mapsto \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix} \mapsto \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ \underline{0} \\ 0 \end{matrix} \mapsto \begin{matrix} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix} \\ \begin{matrix} 0 & 0 & 0 & 0 & 0 & - & 0 \end{matrix} \end{matrix}$$

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