5DV022 **Problem Exercise 3** **Fall 2008**

Solutions to this assignment are due on September 25, 2008 at 1700 (5pm). The signed cover sheet must be turned in with your solutions.

Turn in your solutions to the course instructor. Do not put them in the red mailboxes for laboratory reports.

In this problem, a generalization of the job scheduling problem is considered in which the deadline of a job is replaced by a finite set of admissible slots. Specifically, a *job* job_j has associated with it a *profit* p_j and a finite set $A_j = \{t_{j_1}, \ldots, t_{n_j}\}$ of positive integers, called the set of *admissible slots* for job_j. All jobs have unit running times. The idea is that job_j may only run in the time slots identified by the elements of A_j. Specifically, given a set $\mathbf{J}$ of jobs, a legal *schedule* for a subset I of $\mathbf{J}$ is a function $\sigma : I \to \mathbb{N}^{>0}$ (with $\mathbb{N}^{>0}$ denoting the positive integers) subject to the following constraints.

(i) For any $\mathsf{job}_j \in I$, $\sigma(\mathsf{job}_j) \in A_j$. (Each job is scheduled in one of its admissible slot.)

(ii) For any job_{j_1} and job_{j_2} in I, $\sigma(\mathsf{job}_{j_1}) \neq \sigma(\mathsf{job}_{j_2})$. (Jobs do not overlap.)

An *optimal* schedule is a legal schedule of maximal profit, where the profit of the schedule is the sum of the profits of its constituents. (Notice that the job scheduling problem discussed in the course notes and the text is a special case of this, in which $A_j = \{1, \ldots, d_j\}$, with d_j the deadline of job_j.)

Prove or disprove: The subset system associated with this problem is a matroid. In other words, prove or disprove that the greedy algorithm provides an optimal solution to this job scheduling problem.

Some hints:

- To see how things work, do a few examples before trying to prove anything. Draw pictures of how they are scheduled.
- Note that there is nothing in the definition of a legal schedule (3.3.1 of the lecture slides) which requires that k jobs be scheduled in the first k slots. It is quite legal, for example, to have four jobs, scheduled in time the intervals [2,3], [5,6], [6,7], and [9,10], respectively, with the other slots empty. This also applies to the situation of this problem.
- If the subset system is not a matroid, it suffices to provide a counterexample as the answer.
- If the subset system is a matroid, a productive approach is to consider use 3.2.7 of the lecture slides as a basis for the proof.