

# Graph Transformation for Computational Linguistics

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# Introduction



Traditional grammars and automata used in Computational Linguistics work on **strings**.

- Context-free grammars, linear context-free rewriting systems, multiple context-free grammars, ...

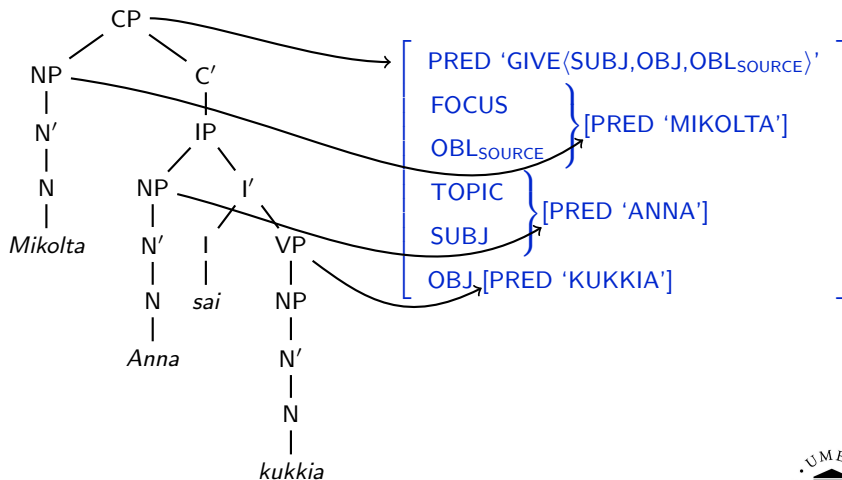
These formalisms were extended to **trees** to be able to handle sentence structure explicitly.

- Regular tree grammars, tree-adjoining grammars, context-free tree grammars, ...

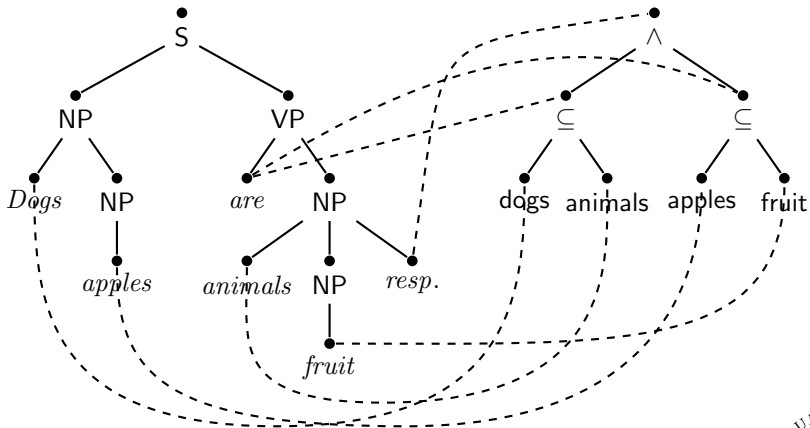
However, in many cases this is not enough.

We rather need to talk about **graphs**.

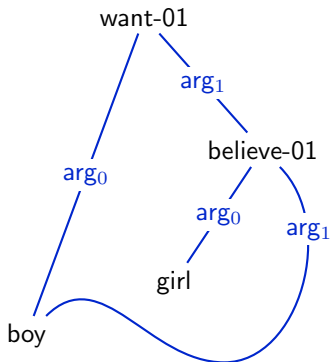
Example (LFG): “Mikolta Anna sai kukkia” [Dalrymple 2001]



Example (Millstream system): "Dogs and apples are animals and fruit, respectively."

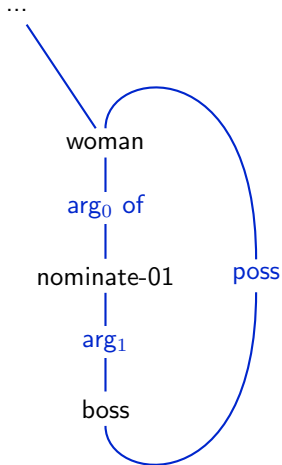


Example (Abstract Meaning Representation): “The boy wants the girl to believe him.” [Banarescu et al. 2014]





Another AMR example: "... the woman who nominated her boss"



Many further examples can be found.

⇒ Computational Linguistics / NLP could benefit from suitable formalisms for generating and transforming graphs.

Such formalisms are provided by the [theory of graph transformation](#) that emerged around 1970.

This talk focusses in particular on [hyperedge replacement grammars](#) [Bauderon & Courcelle 1987], [Habel & Kreowski 1987].

# Graph Transformation

## General idea of graph transformation

- Use rewriting rules  $L \Rightarrow R$  that say “subgraph  $L$  can be replaced by  $R$ ”.
- Similar to string and term rewriting.
- However, how do you insert  $R$  in place of  $L$ ?
- Different answers were given, e.g., **connecting instructions** and **interface graphs** (double-pushout approach, [Ehrig et al. 1973]).

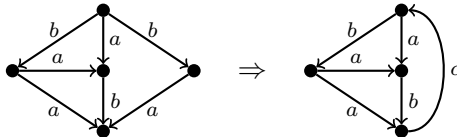
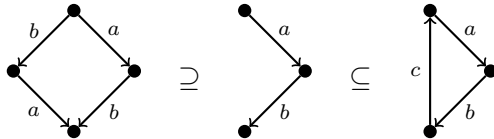
## A few words about the double-pushout approach

- A **rule** is a triple  $r = (L \supseteq I \subseteq R)$  of graphs.
- The middle graph  $I$  is the **interface** between  $L$  and  $R$ .
- To apply  $r$  to a graph  $G$ 
  - ① locate (a copy of)  $L$  in  $G$ ,
  - ② remove  $L - I$ , and
  - ③ add  $R - I$ .

$I$  is the part where old and new items overlap.

# Graph Transformation

## Example



# Context-Free Graph Generation by Hyperedge Replacement



# Hyperedge Replacement

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What would be a suitable **context-free** way of generating graphs?

Idea:

- A derivation step should replace an **atomic item**.
- Two possibilities:
  - replace **nodes** → node replacement grammars
  - replace **edges** → edge replacement grammars



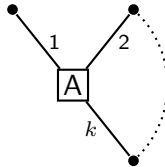
hyperedge replacement grammars



# Hyperedge Replacement

## Hypergraphs

A **hyperedge** with label  $A$  of rank  $k$ :



Ordinary directed edges are included:  $\bullet \xrightarrow{1} \boxed{A} \xrightarrow{2} \bullet$  is  $\bullet \xrightarrow{A} \bullet$

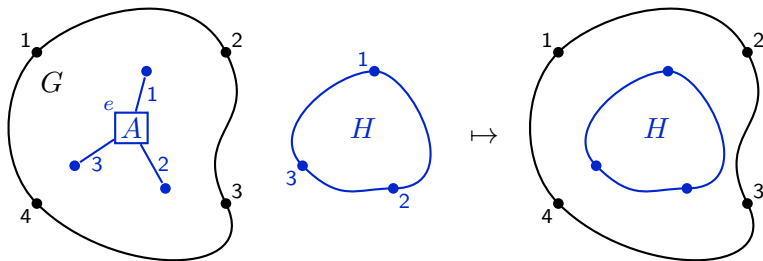
A **hypergraph of rank  $k$**  consists of

- nodes (usually unlabelled),
- hyperedges, and
- a sequence of  $k$  distinguished nodes called **ports**.

## The Replacement Operation

A hyperedge  $e$  of rank  $k$  in a hypergraph  $G$  can be replaced by a hypergraph  $H$  of rank  $k$ :

- 1 build  $G - e$  (remove  $e$  from  $G$ )
- 2 add  $H$  disjointly to  $G - e$
- 3 fuse the  $k$  nodes to which  $e$  was attached with the ports of  $H$ .



## Hyperedge Replacement Grammars

A **hyperedge (HR) replacement grammar**  $\mathcal{G}$  has rules  $A \Rightarrow H$ , where

- $A$  is a nonterminal hyperedge label of rank  $k \in \mathbb{N}$  and
- $H$  is a hypergraph of the same rank  $k$ .

Starting from a start graph, rules are applied until no nonterminal hyperedges are left.

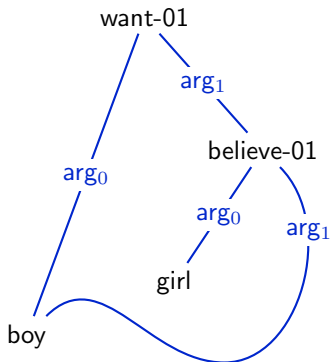
This yields the **language**  $\mathcal{L}(\mathcal{G})$  generated by  $\mathcal{G}$ .

# Hyperedge Replacement

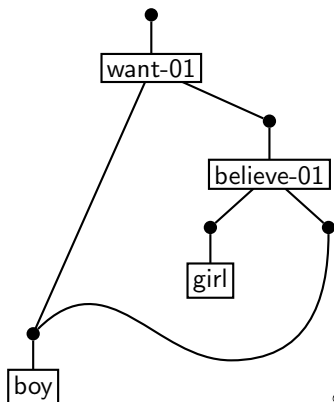
Example: "The boy wants the girl to believe him."

What a derivation could possibly look like.

To be generated:

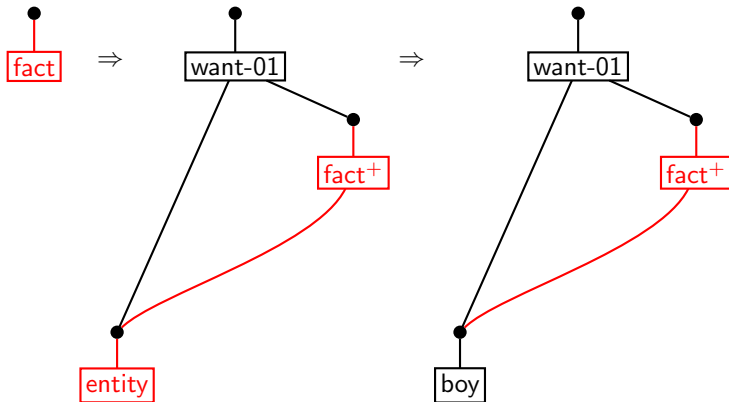


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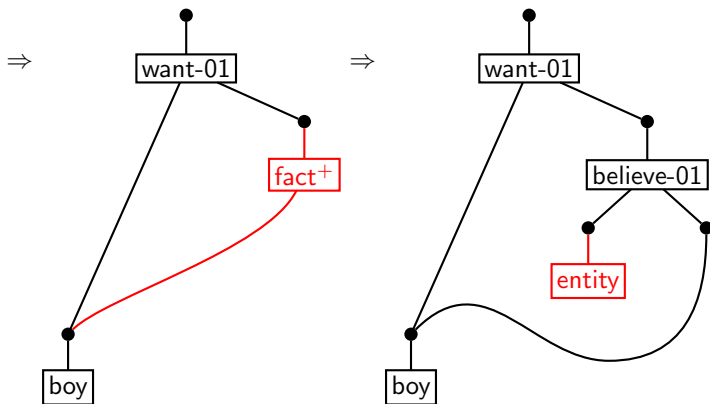
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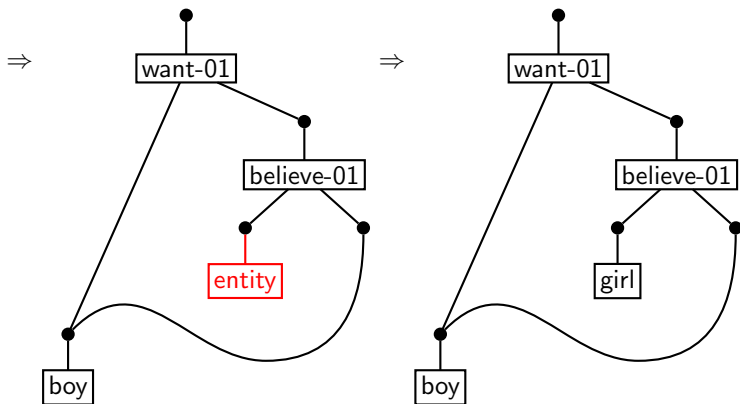
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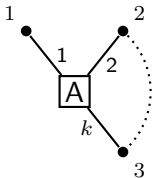
# Some Properties of Hyperedge Replacement Grammars





## Hyperedge replacement is context-free

For a nonterminal symbol  $A$  of rank  $k$  let  $A^\bullet =$



### Context-Freeness Lemma

There is a derivation  $A^\bullet \Rightarrow^{n+1} G$  if and only if there exist

- a rule  $A \Rightarrow H$  and nonterminals  $e_1, \dots, e_k$  with labels  $A_1, \dots, A_k$  in  $H$  and
- derivations  $A_i^\bullet \Rightarrow^{n_i} G_i$

such that  $G = [e_1/G_1, \dots, e_k/G_k]$  and  $n = \sum_{i=1}^k n_i$ .

## Hyperedge replacement and mild context-sensitivity

String languages generated by hyperedge replacement

The string languages generated by HRGs are the same as those generated by

- deterministic tree-walking transducers,
- multiple context-free grammars,
- multi-component TAGs,
- linear context-free rewriting systems,
- ...

[Engelfriet & Heyker 1991]

## Some Properties of HRGs

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- A normal form similar to Chomsky normal-form exists
- Many properties of HR languages are decidable (work by Courcelle, Habel et al., Lengauer & Wanke about inductive/compatible/finite properties)
- Nice connections between HR and monadic second order logic on graphs (though not as perfect as for strings and trees)
- Parikh images are semi-linear (quite obviously, using Parikh's result)

## About parsing

- HR can generate **NP-complete** graph languages [Lange & Welzl 1987]
- **Polynomial** parsing possible in certain cases [Lautemann 1990], [Vogler 1991], [D. 1993], [Chiang et al. 2013]
- Major root of complexity is the large number of ways in which graphs can be decomposed
- Proof by Lange & Welzl very versatile  $\Rightarrow$  rules out many special cases one might otherwise hope to be easier

# Recent Work Aiming at Computational Linguistics and NLP



### Synchronous hyperedge replacement grammars

- [Jones et al. 2012] propose synchronous HRGs for semantics-based machine translation.
- Graphs represent the meaning of a sentence (Abstract Meaning Representations).
- Synchronous HRGs are defined “as one would expect”.
- These are further extended by probabilities and then trained.

**Note:** Even though nobody seems to have mentioned it anywhere, synchronous HRGs are simply HRGs.

## Lautemann's Parsing Algorithm Revisited

- [Chiang et al. 2013] refine Lautemann's parsing algorithm.
- Aim: Improve its efficiency in practice, so that it can be used in NLP settings.
- Main novelty: Use tree decompositions in order to reduce the search space.

### Bolinas

- Bolinas is a software toolkit implementing synchronous HRGs, parsing, training, and other relevant algorithms.
- Developed at USC/ISI, see [Andreas et al. 2013].
- Implemented in Python.



## Readers for incremental sentence analysis

- [Bensch et al. 2014] use graph transformation to turn a sentence into a graph that represents its analysis.
- Each word  $w$  is associated with a set  $\Lambda(w)$  of rules.
- A derivation

$$G_0 \xRightarrow{\Lambda(w_1)} G_1 \xRightarrow{\Lambda(w_1)} \cdots \xRightarrow{\Lambda(w_n)} G_n$$

is a **reading** of  $w_1 \cdots w_n$  that yields the analysis  $G_n$ .

- Soundness w.r.t. a so-called **Millstream system** turns out to be decidable.

A **prototype implementation** by F. Rozenberg under the supervision of H. Jürgensen (Western University, Ontario) is underway.



## Some More Questions for Future Work



### Efficient parsing for cases that occur in Computational Linguistics?

- A typical type of structures that occurs in Computational Linguistics are directed acyclic graphs (DAGs).
  - In general, HR languages of dags can be NP-complete (easy adaptation of the proof by Lange & Welzl).
- ⇒ Aim: identify cases not covered by known parsing algorithms, in which parsing is nevertheless “easy” .

### From strings to graphs

- The input to an NLP task is often a sentence.
- An analysis of the sentence is a graph (cf. LFG, HPSG, AMR).

How do we get from one to the other?

Readers are a first attempt, but further techniques must be explored.

## Implementations?

- **Bolinas** implements HRGs and its algorithms from scratch.
- General purpose implementations of graph transformation have existed for quite a while.
- In particular, GrGen [Geiß et al. 2006] (now GrGen.net [Jakumeit et al. 2010]) is very efficient and powerful.

⇒ Question: Can systems such as GrGen.net be of interest as a basis for implementations?

**Note:** F. Rozenberg's implementation of readers uses GrGen.net.

## Some More Questions

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There is a lot to do;  
most questions have not even been asked yet.

Please join if you are interested.

Thank you!





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